

**LABASA SANGAM (SKM) COLLEGE**  
**YEAR 13 MATHEMATICS-WORKSHEET**

**WEEK 1**

**TOPIC: VECTORS**

1. Points A and B have position vectors  $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ .

(i) Express the vector AB in terms of unit vector.

(ii) Determine the scalar product of  $\vec{a}$  and  $\vec{b}$

(iii) Find the angle between  $\vec{a}$  and  $\vec{b}$ .

2. Write down the symmetric equation of the line passing through the point  $(2, -1, 3)$  in the direction of  $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ .

3. If P is a point  $(1, 1, 0)$  and R is the point  $(1, 6, -5)$  find the coordinates of point Q on line PR given that  $PQ:QR=3:2$ .

4. The position vectors  $\vec{a}$  and  $\vec{b}$  are defined by:

$\vec{a} = i + 2k$  and  $\vec{b} = i - 3j + k$ . Find:

(i)  $|\vec{a}|$       (ii)  $|\vec{b}|$       (iii) the scalar product  $\vec{a} \cdot \vec{b}$

5. Write the symmetric equation of the line passing through the point  $(1, -2, -4)$  in the direction of  $\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$ .

6. If M is a point  $(3, 0, -1)$  and N is the point  $(-2, -1, 5)$ , find the coordinates of point P on line MN given that  $MP:NP=4:-3$ .

7. Let  $\vec{a} = 4i - 2j - 4k$  and  $\vec{b} = 3j - 6k$ .

Find  $\vec{a} - \vec{b}$ .

8. Find the unit vector that has the same direction

as  $\vec{v} = 2i - j - 2k$ .

9. Find the parametric equations of the line passing the points  $(3, 4, -1)$  and  $(9, 0, 7)$ .

10. Let  $\vec{a} = \begin{pmatrix} 4 \\ k \\ 7 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ . Find the value of  $k$  if  $\vec{a}$  and  $\vec{b}$  are orthogonal.

## WEEK 2

### TOPIC : COMPLEX NUMBERS

1. A complex number is given as  $Z = 2 + 3i$ , Find

(a)  $\text{Re}(Z)$  (b)  $\text{Im}(Z)$  (c)  $\bar{Z}$  (d)  $Z + \bar{Z}$

2. Use the quadratic formula to solve  $x^2 - 10x + 26 = 0$  where  $x \in \mathbb{Z}$ .

3. A complex number is given as  $w = 1 + \sqrt{3}i$

(i) Find  $\text{Arg}(w)$  (ii) Find  $|w|$

(iii) Convert  $w$  into polar form

(iv) Hence, evaluate  $w^4$  using De Moivre's Theorem

4. Solve the equation  $z^3 = 216(\cos 60 + i \sin 60)$  obtaining three distinct complex roots. Leave your answers in polar form.

5. Evaluate  $\sqrt{-100}$

6. Complex numbers  $u$  and  $v$  are given as

$$u = 3(\cos 90 + i \sin 90)$$

$$v = 5(\cos 180 + i \sin 180)$$

Find  $uv$ .

7. Express  $\frac{13}{3 + 2i}$  in the form  $a + bi$

8. Let  $w = \sqrt{8} + \sqrt{8}i$

(i) Find  $\text{Arg}(w)$  (ii) Find  $|w|$

(iii) Convert  $w$  into polar form

(iv) Use De Moivre's Theorem to evaluate  $w^6$ .

9. State whether each of the following statement is true or false.

(a)  $i^2 + i^4 = 0$  (b)  $2\text{cis}20 \times 3\text{cis}15 = 6\text{cis}100$

(c) If  $\text{Arg}(Z) = 0$  then  $\text{Arg}(\bar{Z}) = 180$

(d) Region represented by  $|z| \leq 3$  is inside the circle having center  $(0,0)$  and radius 3 units.

10. Find the values of  $x$  and  $y$  such that  $x + yi = \sqrt{4} - \sqrt{-9} + \sqrt{-16}$

### WEEK 3

#### TOPIC: PROBABILITY AND INFERENTIAL STATISTICS

1. Two fair dice are rolled and the numbers on the uppermost are observed. Let event A = the number on the first dice is 5, event B – the sum of the numbers is more than 9. Workout the following probabilities:

(a) P(A)                      (b) P(B)                      (c) P(A ∩ B)

2. Using the formula  $n = \left( \frac{Z_{\frac{\alpha}{2}} \sigma}{e} \right)^2$

Calculate the minimum sample size that should be taken from a population with standard deviation of

0.7kg to estimate the mean weight to be within 0.1 of the true value with 98% confidence.

3. X has a normal distribution with mean =20. Given that P(X>30)=0.1. Calculate the standard deviation.

4. 10% of items produced by a certain machine are defective. Determine the probability that out of sample of 7 randomly chosen items exactly 3 are defective.

5. A dalo farmer claims that the average weight of a bundle of dalo which he sells is 9kg with standard deviation of 0.5kg. Test the null hypothesis  $H_0 : \mu = 9$  against the alternative hypothesis  $H_A : \mu < 9$  if a random sample of 49 bundles has an average weight of 8kg. Use a 0.05 level significance and state your conclusion clearly.

6. A sample of 100 items is taken from a population with unknown mean  $\mu$  and standard deviation 8. The sample mean is 500.

Using the formula  $\bar{x} - Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

or otherwise, find a 90% **confidence interval** for  $\mu$

$\square\square$  The probability of Anna passing Accounting (event A) is 0.7, while the probability that she passes Economics (event B) is 0.8. The probability that she passes **both** is 0.6, that is  $P(A \cap B) = 0.6$

(i) Why are events A and B **not** mutually exclusive?

(ii) Find  $P(A \cup B)$ , the probability that event A **or** B occurs.

8. The probability that a bean seed **germinates** is 0.2. A farmer plants 8 bean seeds.

(i) State two ways in which the above situation meets the **Binomial Distribution** conditions.

(ii) Using the Table of Binomial Distribution or otherwise, find the probability that **exactly 3** seeds germinate?

9. A goat farmer weighed a random sample of 60 goats from his farm. He wanted to test the claim that the mean weight of goats more than one year old in a goat farm is 20 kg with a standard deviation of 4 kg.

(i) Construct a test at 1% significance to determine whether the null hypothesis,

$H_0 : \mu = 20$  kg can be accepted given the alternative hypothesis,  $H_A : \mu \neq 20$  kg.

(ii) If the sample mean is 22 kg, what is your conclusion?

## WEEK 4

### TOPIC: FUNCTIONS

1. Sketch the graph of the polynomial shown below. Clearly show the y-intercept, x-intercepts, turning points and the point of inflection.

$$y = \frac{-1}{4}(x-1)^3(x+1)^2(x-4)$$

2. Consider the top heavy function  $f(x) = \frac{x^2 + x}{-2x + 2}$ .

- (a) Find the x and y intercepts.
- (b) Find the equation of the vertical asymptote.
- (c) Determine the equation of the oblique asymptote.
- (d) Hence, sketch the graph of the function  $f(x)$ .

3. Let  $f(x) = 7 - x^2$  and  $g(x) = \sqrt{x-1}$ .

- (a) Find an expression for  $f \circ g(x)$ .
- (b) What is the range of  $f \circ g(x)$ ?

4. The functions f and g are defined by

$$f(x) = x^2 \text{ and } g(x) = \sqrt{x-4}$$

- (a) Find an expression for  $f \circ g(x)$ . Simplify your answer
- (b) State the domain of  $f \circ g(x)$

5. Sketch the graph of  $y = (x-1)(x+1)^3(x-3)^2$ , Clearly showing all the intercepts, turning point and point of inflection.

6. Consider the function  $f(x) = \frac{(x+1)(x+4)}{(x-2)(x+2)}$ .

- (i) Find the x and y intercepts of the graph of  $f(x)$ .
- (ii) State the equations of the asymptote of  $f(x)$ .
- (iii) Hence, sketch the graph of the function  $f(x)$ .

7. Sketch  $f(x) = \frac{(x+1)}{(x-2)(x+2)}$