

BA SANGAM COLLEGE

YEAR 13

MATHEMATICS

WORKSHEET 1

STRAND 1 – COMPLEX NUMBERS

- 1) Given that $Z = 2 + i$
a) Write Z in polar form

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- b) Show that $Z = |Z|^2$

Z

- 2) Solve: $4x^2 + 9 = 0, x \in \mathbb{Z}$

- 3) Express $\frac{3+2i}{1-4i}$ in the form $a + bi$

- 4) For the complex number
i. Write Z in polar form
ii. Find using De Moivre's Theorem and express the answer in **Rectangular form**

- 5) The Polar form of a complex number Z is given as $64\left(\cos \frac{-\pi}{3} + i \sin \frac{-\pi}{3}\right)$ Find the cube roots of Z and express your answers in **rectangular form**

- 6) Two complex numbers are given as $U = -3 + 4i$ and $V = 5 - 12i$. Show that $|UV| = |U| |V|$.

- 7) Given $P = 5(\cos 90^\circ + i \sin 90^\circ)$ and $Q = 2(\cos -30^\circ + i \sin -30^\circ)$,
a) Represent P on an Argand diagram.
b) Find PQ and express your answer in rectangular form.

8) Express $\frac{2+5j}{i}$ in the form of $a+bi$

9) A complex number Z is given as i

a) Express Z in polar form.

b) Use De Moivre's Theorem to find Z^5 , in **rectangular form**.

10) Find the three distinct roots of the complex equation $W^3 = 8(\cos 270^\circ + i \sin 270^\circ)$

STRAND 2 - VECTORS

1. The position vector \vec{a} and \vec{b} are defined by:

$$\vec{a} = i + 2k \text{ and } \vec{b} = i - 3j + k \text{ Find :}$$

b

i. $|\vec{a}|$ and $|\vec{b}|$

ii. Express the vector \vec{AB} in terms of the unit vectors i, j and k

iii. Find the angle between \vec{a} and \vec{b}

2. Given that \vec{a} and \vec{b} , find the constant k such that

3. The vector equation of a line is given as $\vec{r} = \vec{a} + t\vec{b}$. Write the symmetric equation of this line.

4. If R and S are the points $(-1, 2, 4)$ and $(3, 0, -2)$ respectively, find the coordinates of point P such that $\frac{RP}{PS} = \frac{-1}{3}$
5. Given that \mathbf{a} and \mathbf{b} are vectors
- Find $\mathbf{a} - 2\mathbf{b}$
 - Show that the two vectors \mathbf{a} and \mathbf{b} are orthogonal
6. Two points are given as $P_1(-2, 3, 5)$ and $P_2(3, -1, -2)$.
Find the equation of the line passing through point P_1 in the direction P_1P_2 .
7. For the equation of the line P_1P_2 joining the points $P_1(-2, 3, 5)$ and $P_2(3, -1, -2)$ in
- Vector form.
 - Parametric form.
 - Symmetric form

STRAND 3 - FUNCTIONS

1. The functions $f: x \rightarrow x^2 + 1$ and $g: x \rightarrow 4x - 2$

Find:

- $f - g(x)$
- domain of $f - g(x)$
- $f \circ g(x)$

2. The functions f and g are defined by $f(x) = \dots$ and $g(x) = \dots$

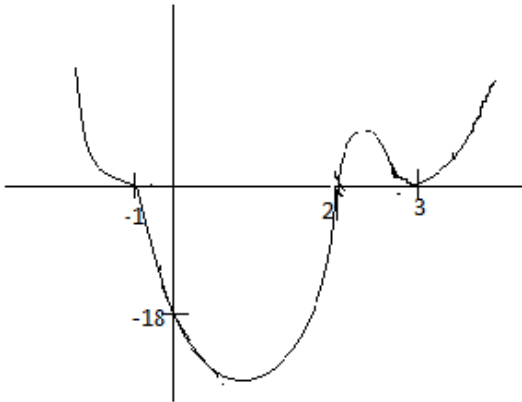
Find

- $f \circ g(x)$
- $f + g(x)$
- the domain of $f \circ g(x)$
- the range of $f \circ g(x)$

3. Sketch the graph of $y = (x - 1)^2(x + 2)^3$

(Clearly show all the intercepts, turning points and the point of inflection)

4. Write the equation for the polynomial function shown in the graph below.



5. The graph of a rational function, $g(x)$, has the equation $g(x) = \frac{x^2 + 4x + 3}{x - 1}$.
- Find the x and y intercepts of $g(x)$.
 - Find the equation of the vertical asymptote of $g(x)$.
 - Find the equation of the oblique asymptote of $g(x)$.
 - Sketch the graph of $g(x)$, showing the intercepts and the asymptotes.

6. A rational function is given by $g(x) = \frac{3-x}{(x+1)(x-2)}$
- Find the x and y intercepts of the graph of $g(x)$.
 - Identify the asymptotes and give their equations.
 - Sketch the graph of the function $g(x)$ clearly showing the intercepts and asymptotes.

7. The graph of a rational function, $g(x)$, has the equation

$$g(x) = \frac{(x+1)(x-3)}{(x+2)(x-2)}$$

- Find the x and y intercepts of the graph of $g(x)$.
 - Find the equation(s) of the vertical asymptote(s) of the graph of $g(x)$.
 - Find the equation of the horizontal asymptote of the graph of $g(x)$.
 - Sketch the graph of $g(x)$, showing the intercepts and asymptotes.
8. A polynomial function is given as $y=f(x)$.

Given that the function satisfies $f(0)=f(2)=f(5)=0$ and $f(3)=24$,

- State the x and y intercepts of the function.
- Find the equation of the function.
- Sketch the graph of the function

STRAND 4 - LIMITS AND CONTINUITY

1. Find

$$\frac{4 - 7x - 3x^2}{x + 7}$$

a.

$$\frac{4 - 7x - 3x^2}{x + 7}$$

b.

2. Evaluate the following limits:

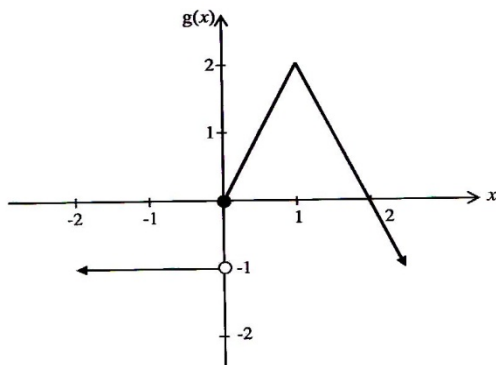
$$\frac{(2x + 5)^2}{4 - x^2}$$

a.

$$\frac{(2x + 5)^2}{4 - x^2}$$

b.

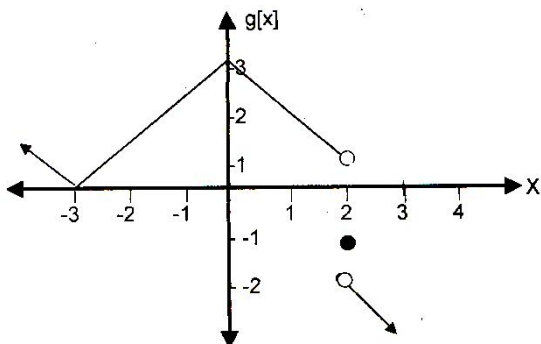
3. The graph of a function $g(x)$ is shown below.



Use the graph above to find the value(s) of x for which $g(x)$ is:

- a. discontinuous
- b. non-differentiable
- c. equal to zero

4. The graph of another piece-wise function $g(x)$ is given below.



- a. For what value(s) of x is $g(x)$ discontinuous?
- b. For what value(s) of x is $g(x)$ non-differentiable?

c. Find $\lim_{x \rightarrow 2} g(x)$

d. Find $\lim_{x \rightarrow 2^-} g(x)$

e. Find $g(2)$