SUVA SANGAM COLLEGE
YEAR 12
MATHEMATICS
WEEK 1: MONDAY 05/07 TO FRIDAY 09/07

STRAND	12.2 Algebra
SUB-STRAND	12.2.1 Equations and In-equations
CONTENT LEARNING OUTCOME	12.2.1.4 Solve quadratic equations and in – equations
REFERENCE FROM TEXTBOOK	Pg 58

Achievement Indicators
1. Solve quadratic equations and in – equations

Notes

Steps to solve
1. To solve equations, follow steps of making the variable, preferably 'x' the subject.
2. If you multiply or divide each side by a negative quantity, the inequality symbol must be reversed.
3. Sketch the graph
4. Follow the signs

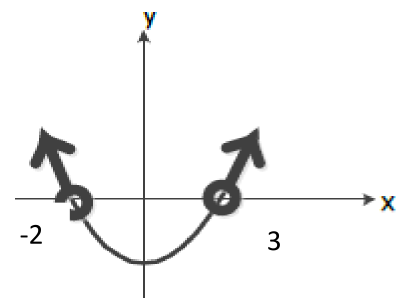
$<, \leq$ Highlight below the x-axis
$>, \geq$ Highlight above the x-axis
(open circle −for $<, >$)
(closed circle-for $\leq, \geq$)

Example 1
Solve $x^2 - x - 6 > 0$

$x^2 - x - 6 > 0$
$(x+2)(x-3) > 0$

x $intercepts$
$x + 2 > 0, x - 3 > 0$
$x > -2$, $x > 3$

So, place open circle and highlight the region above the x -axis.

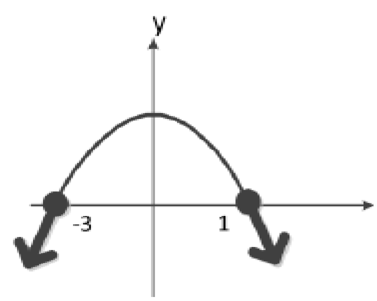


$\therefore \boldsymbol{x < -2 \text{ or } x > 3}$

Example 2
Solve $x^2 + 2x - 3 \leq 0$

$x^2 + 2x - 3 \leq 0$
$(x+3)(x-1) \leq 0$
x $intercepts$
$x + 3 \leq 0, x - 1 \leq 0$
$x \leq -3$, $x \leq 1$

So, place closed circle and highlight the region below the x -axis.



$\therefore \boldsymbol{x \leq -3 \text{ or } x \geq 1}$

Student Activity

1. Solve $x^2 + 7x + 12 \geq 0$

2. Solve $x^2 - 4x - 5 < 0$

3. Solve $x^2 > x$

WEEK 2: MONDAY 12/07 TO FRIDAY 16/07

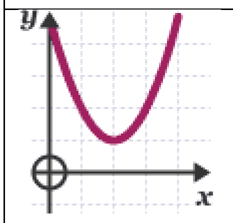
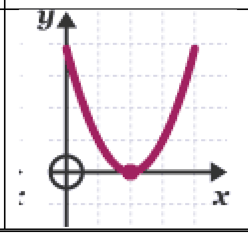
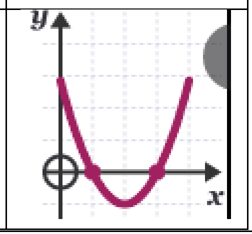
Achievement Indicators

i. Find discriminants

ii. State the nature of roots

NOTES

- The Discriminant of a quadratic equation $D = b^2 - 4ac$.
- The quadratic equations should always be in the form $ax^2 \pm bx \pm c = 0$

Case 1	Case 2	Case 3
discriminant is zero $b^2 - 4ac < 0$	discriminant is negative $b^2 - 4ac = 0$	discriminant is positive $b^2 - 4ac > 0$
No real root	One repeated real roots	Two distinct real roots
		

Example 1

Find the value of the discriminant of the equation $2x^2 - 3x + 5 = 0$ and state its nature of roots.

Solution

$2x^2 - 3x + 5 = 0$ compare the equation with

$ax^2 \pm bx \pm c = 0$

$a = 2, \quad b = -3, \quad c = 5$

We know that

$D = b^2 - 4ac.$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31$

Since $D > 0$, there is no real roots

Concept
$D < 0, No\ real\ roots$
$D = 0, One\ repeated\ real\ root$
$D > 0, Two\ distinct\ real\ roots$

Example 2

A quadratic equation is given as $-4x^2 + kx - 1 = 0$ .Find the value (s) of k such that $-4x^2 + kx - 1 = 0$ has only 1 real root?

$-4x^2 + kx - 1 = 0$

$a = -4, \quad b = k, \quad c = -1$

$D = b^2 - 4ac.$

$b^2 - 4ac = 0$

$(k)^2 - 4(-4)(-1) = 0$

$(k)^2 - 16 = 0$

$(k)^2 = 16$

$k = \sqrt{16}$

$k = \pm 4$

Example 3

Find the value of the discriminant of the equation $\frac{x^2}{2} - x - 2 = 0$ and state its nature of roots?

Solution

$\frac{x^2}{2} - x - 2 = 0$

Compare hence a = ½ , b = −1 and c = −2

Substitute in the formula

$D = b^2 - 4ac$

$D = (-1)^2 - 4\left(\frac{1}{2}\right)(-2)$

$D = 5$

Since,$D > 5$, there are two real roots.

Student activity

FY12CE 2018

1. Consider the quadratic equation
   $3x^2 - 4x + 5 = 0$

   a) Calculate the value of the **discriminan**t.

   b) Hence, state the **nature** of the roots.

FY12CE 2017

2. A quadratic equation is given as
   $2x^2 + 3x + 1 = 0$

   a) Calculate the value of the **discriminant**.

   b) Hence, state the **nature** of the roots.

FY12CE 2016

3. A quadratic equation is given as $4x^2 + 3x + p = 0$. Find the value(s) of p such that $4x^2 + 3x + p = 0$ has 2 distinct real roots.

WEEK 3: MONDAY 19/07 TO FRIDAY 23/07

Achievement Indicators

- Simplify algebraic expressions

Use BEDMAS rule to simplify

B	Brackets	( )
E	Exponents / powers	x^2
D	Division	÷ whichever comes first
M	Multiplication	x
A	Addition	+ whichever comes first
S	subtraction	-

Rules for dividing fractions

- Change the division sign to multiplication
- Take the reciprocal (swap the numerator and the denominator) of the next term
- Multiply and simplify

Example 1

Simplify $\frac{2x}{y} - \frac{4x^2}{3y} \div \frac{8x}{9}$

$\frac{2x}{y} - \frac{4x^2}{3y} \div \frac{8x}{9}$	Change the ÷ sign to × and take the reciprocal of the second fraction
$\frac{2x}{y} - \left(\frac{4x^2}{3y} \times \frac{9}{8x}\right)$	Collect the numerator and denominator separately (direct multiply)
$\frac{2x}{y} - \frac{36x^2}{24xy}$	Simplify the fraction (cancel the common factors)
$\left(\frac{2x}{y}\right) \times 2 - \frac{3x}{2y}$	Find the lowest common Denominator and collect like terms
$\frac{4x}{2y} - \frac{3x}{2y}$	Since the denominator is same, simplify the numerator
$\frac{4x - 3x}{2y}$	Collect like terms
$\frac{x}{2y}$	

Example 2

Simplify $\frac{a^2-b^2}{8} \div \frac{b^2+ab}{8a+8}$

$\frac{a^2-b^2}{8} \div \frac{b^2+ab}{8a+8}$	Change the ÷ sign to × and take the reciprocal of the second fraction
$\frac{a^2-b^2}{8} \div \frac{8a+8}{b^2+ab}$	Factorize the individual algebraic expressions
$\frac{(a+b)(a-b)}{8} \times \frac{8(a+1)}{b(a+b)}$	Cross off the common factors
$\frac{(a-b)}{1} \times \frac{(a+1)}{b}$	Simplify the fraction
$\frac{(a-b)(a+1)}{b}$	

Example 3

Simplfy $\frac{x^2-7x+12}{x^2-16}$

$\frac{x^2-7x+12}{x^2-16}$	Factorize the algebraic fractions - Numerator quadratic type 1 method - Denominator difference of 2 squares
$\frac{(x-3)(x-4)}{(x-4)(x+4)}$	Cancel the common factors
$\frac{x-3}{x+4}$	

Student Activity

FY12CE 2017

1. Simplify $\frac{4x}{y} - \frac{x}{3} \div \frac{y}{3}$

FY12CE 2016

2. Simplify $\frac{x^2+2x}{8} \div \frac{x+2}{16}$

3. Simplify $\frac{5x}{3} - 4y \div \frac{12y}{7x}$

WEEK 4: MONDAY 26/07 TO FRIDAY 30/07

STRAND	12.2 Algebra
SUB-STRAND	12.2.2 Remainder and Factor Theorem
CONTENT LEARNING OUTCOME	12.2.2.1 Study and work with Cubic expressions
REFERENCE FROM TEXTBOOK	Pg 65 - 67

Achievement Indicators

- Show that a given expression is a factor
- Calculate the remainders.

Remainder theorem: if a polynomial $p(x) is\ divided\ by\ the\ binomial\ x-a$, the remainder obtained is p(a).

Example

Consider a divisor $(x-a)$ which is also a factor of a polynomial $f(x)$, Then

Steps to find the remainder

1. Find the value of x i.e. by solving
   $divisor = 0$
   $x - a = 0$
   $x = a$
2. Substitute the x value in the polynomial $f(a)$.
3. The remainder = $f(a)$.

When any polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$

Example 1

A function is given as $f(x) = x^3 - 2x^2 - 5x + 6$
Show that $(x + 2)$ is a factor of $f(x)$

find the value of x by equating to zero	$x + 2 = 0$ $x = -2$
Substitute the value of x in the polynomial $f(x)$	$f(x) = x^3 - 2x^2 - 5x + 6$ $f(x) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$ $f(x) = -8 - 8 + 10 + 6$ $f(x) = 0$
Since $f(-2) = 0, (x + 2)$ is a factor of $f(x)$.	

Example 1

Calculate the remainder when $f(x) = x^2 - 2x + 1$ is divided by $x - 2$.

Find the value of x by equating divisor to 0	$x - 2 = 0$ $x = 2$
Substitute the x value in the polynomial $f(a)$	$f(x) = x^2 - 2x + 1$ $f(2) = (2)^2 - 2(2) + 1$ is $f(2) = 4 - 4 + 1$ $= 1$
The remainder is equal to f(a)	$f(2) = 1$

Example 2

Determine the value of ***a*** if
$f(x) = x^3 - 2x^2 + 3ax + 6$ has a remainder of 7 when divided by
$x - 1$.

Find the value of x by equating divisor to 0	$x - 1 = 0$ $x = 1$
Substitute the value	$f(x) = x^3 - 3x^2 + 3x - 6$ $7 = (1)^3 - 3(1)a^2 + 3(1) - 6$ $7 = 1 - 3a + 3 - 6$ $7 = -3a - 2$ $9 = -3a$ $-3 = a$

STUDENT ACTIVITY

FY12CE 2015

1. A polynomial function is given by

$f(x) = x^3 + kx^2 - 5x - 6$

Find the value of k if $(x - 2)$ is a factor of $f(x)$.

FY12CE 2018

2 Calculate the value of k given that $x + 2$ is a factor of

$f(x) = x^3 + 8x^2 + 17x + k$

3. A function $f(x)$ is given as

$f(x) = x^3 + x^2 - 17x + 15.$

Show that $(x - 1) is\ a\ factor\ of$ $f(x)$.

WEEK 5: MONDAY 02/08 TO FRIDAY 06/08

Achievement Indicators

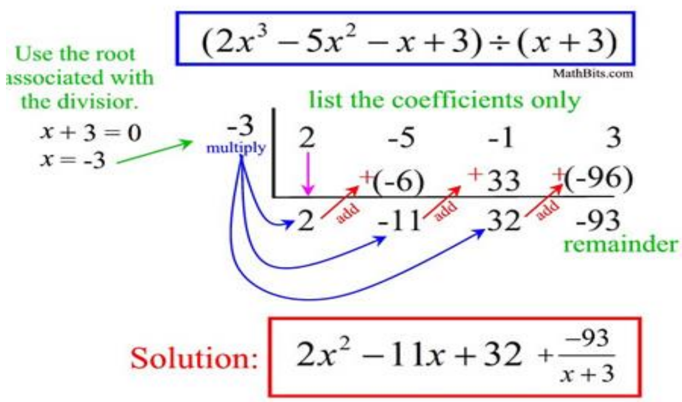
- Factorize cubic Expressions

Synthetic division is another way to divide a polynomial by the binomial x - c , where c is a constant.
Step 1: Set up the synthetic division. ...
Step 2: Bring down the leading coefficient to the bottom row.
Step 3: Multiply c by the value just written on the bottom row. ...
Step 4: Add the column created in step 3.

Example 1

Use synthetic method to divide
$f(x) = 2x^3 - 5x^2 - x + 3$ by $x + 3$.

Solution



Example 2

Use the synthetic method factorize
$f(x) = x^3 + 2x^2 - 5x - 6$ by $x - 2$.

the value of x by equating divisor to 0	$x - 2 = 0$ $x = 2$
Use synthetic method to divide	2 | 1 2 −5 −6 2 8 6 1 4 3 0 $x^2 + 4x + 3$
Factorize completely Use quadratic trinomial method of factorization	$x^2 + 4x + 3$ $(x - 2)\ (x + 1)(x + 3)$

Example 3

Use the synthetic method factorize
$f(x) = x^3 + 2x^2 - 13x + 10$ *by* x +5

Find the value of x by equating divisor to 0	$x + 5 = 0$ $x = -5$
Use synthetic method to divide	-5 | 1 2 − 13 10 -5 15 10 1 -3 2 0 $x^2 - 3x + 2$
Factorize completely	$x^2 - 3x + 2$ $(x - 1)(x - 2)(x + 5)$

STUDENT ACTIVITY

FY12CE 2016

1. A polynomial function is given by
   f(x)=$x^3 - 5x^2 - 2x$ +24
   Given that $x + 2$ is one of the factors of $f(x)$, find the other two factors.

FY12CE 2017

2. A polynomial function is given by
   $f(x) = x^3 - x^2 + x + 6$

   Given that (x-3) is one of the factors of , find the other two factors.

FSLC 2013

3. A function $f(x)$ is given as
   $f(x) = x^3 + 2x^2 - kx - 6.$

(a) Find the value of k if $(x + 3)$ is a factor of $f(x)$.

(b) Factorize $f(x)$ completely.

WORKSHEET

1.	The remainder when $x^3 + 4x^2 - x + 3$ is divided by $(x + 2)$ is A. -19 B. 3 C. 13 D. 25
2.	If $f(x) = x^2 - 3x + 2$, the value of $f(2)$ is A. -1 B. 0 C. 1 D. 4
3.	Simplify $\dfrac{x^2 + 2xy}{2x^3y} \div \dfrac{xy + 2y^2}{6x^2y^3}$
4.	A quadratic equation is given as $2x^2 + 3x + 1 = 0$. (i) Calculate the value of the **discriminant**. (ii) Hence, state the **nature** of the roots.
5.	A polynomial function is given by $f(x) = x^3 + kx^2 - 5x - 6$. (i) Find the value of k if $x-2$ is a factor of $f(x)$. (ii) Hence factorise $f(x)$ completely.
6.	A quadratic equation is given as $x^2 - px + 4 = 0$. Find the values of p such that $x^2 - px + 4 = 0$ has **real roots**. (Hint: Solve $b^2 - 4ac \geq 0$).