SUVA SANGAM COLLEGE
YEAR 13
MATHEMATICS

WEEK 1: MONDAY 05/07 TO FRIDAY 09/07

Strand	Limits, Continuity and Differentiability
Sub – Strand	Limits
Content Learning Outcome	Explore the method of finding Limits
Reference from Text Book	Pg 108 - 115

Achievement Indicator:

- To calculate limits. Using substitution and Table method

General Formula

$$\lim_{x\to a} f(x) = f(a)$$

limit

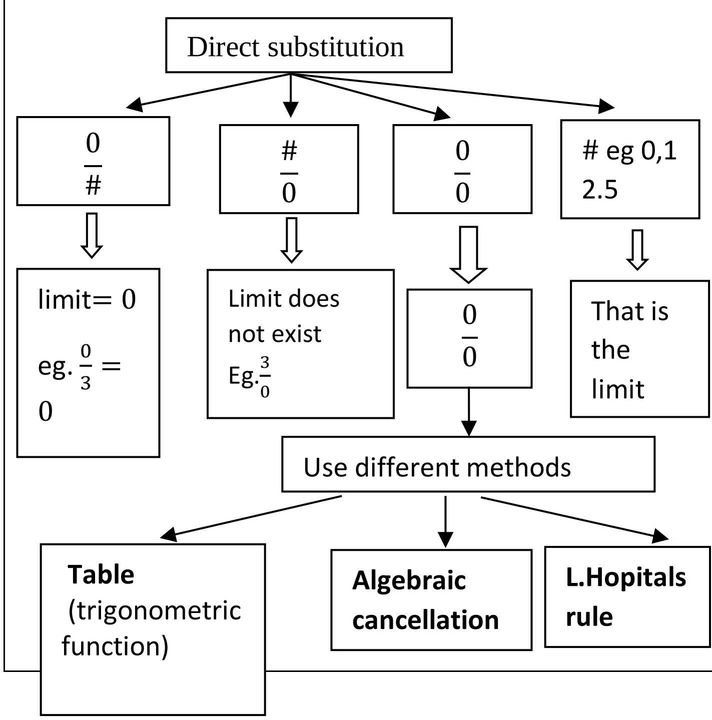
approaches function

Steps

- Use direct substituition.
- Substitute the value(a) directly into the expression.

Case 1: x approaches a number

Upon direct substitution, the result will be one of the following:



Type 1: Table Method

Example 1

Find the limit

$$\lim_{x\to 0}\frac{x^2+x}{x}$$

Solution

Direct substitution

$$\lim_{x\to 0}\frac{x^2+x}{x}$$
$$=\frac{o^2+0}{0}$$
$$=\frac{0}{0}$$
$$\therefore \ limit\ exists$$

The value gets closer and closer to 1 as x approaches 0

Table Method

$f(x)$	-0.01	0	0.01
$\frac{x^2+x}{x}$	$\frac{(-0.01)^2+(-0.01)}{(-0.01)}$ $=0.99$	$-$	$\frac{(0.01)^2+(0.01)}{(0.01}$ $=1.01$

$\longrightarrow$ 1 $\longleftarrow$

$$\therefore \lim_{x\to 0}\frac{x^2+x}{x}=1$$

Example 2

Find $\lim_{x\to 0}\sin x$

Direct substitution

$sin\ 0 = 0$

(in this case the limit is zero.)

$\therefore \lim_{x\to 0}\sin x = \mathbf{0}$

Example 3

Find $\lim_{x\to -5}\frac{x+8}{x-2}$

Direct substitution

$\frac{-5+8}{-5-2}=\frac{3}{-7}$ $\therefore \lim_{x\to -5}\frac{x+8}{x-2}=\frac{-3}{7}$

Example 4

Find $\lim_{x\to 3}\frac{2x^2-5x-8}{x-3}$

Direct substitution

$$=\frac{2(3)^2-5(3)-8}{3-3}=-\frac{5}{0}$$

In this case **limit does not exist**

Trigonometric Functions (sine, cosine, tangent)

- Logarithmic and ln functions are not trigonometric

Note: when using trigonometric function
Calculator to be in radian mode

Steps: PRESS | Mode | → | Mode | → | 2 |

After doing the question change the calculator to degree mode.

Steps: | mode | | mode | | 1 |

Example:

Find $\lim_{x \to 0} \frac{\sin x}{x}$

Solution

Direct substitution

$\lim_{x \to 0} \frac{\sin x}{x}$

$= \frac{\sin 0}{0} = \frac{0}{0} \therefore \textit{limit exists}$

Therefore use a **table method**

Calculator to be in radian mode

x	-0.01	0	0.01
$\frac{\sin x}{x}$	$\frac{sin(-0.01)}{(-0.01)}$ $= 0.99$	–	$\frac{\sin(0.01)}{0.01}$ =0.99

$\longrightarrow$ 1 $\longleftarrow$

The values get closer and closer to 1 as x approaches zero.

$\therefore \lim_{x \to 0} \frac{\sin x}{x} = 1$

STUDENT ACTIVITY

Calculate the following limits

(a) $\lim_{x \to 0} \frac{\sin x}{x}$

(b) $\lim_{x \to 0} \frac{tanx}{sin2x}$

(c)

Calculate the value of $\lim_{x \to 0} 3e^x + 1$

WEEK 2: MONDAY 12/07 TO FRIDAY 16/07

Achievement Indicator:

- To calculate limits when x approaches infinity.

Case 1: x approaches infinity(∞)

Type 1

$$f(x) = \frac{ax+b}{cx+d}$$

Type 2

$$f(x) = a \pm \frac{bx}{cx+d}$$

Cross multiply

$$f(x) = \frac{a}{1} \pm \frac{bx}{cx+d}$$

Steps

- For the rational function in the form $\frac{ax+b}{cx+d}$
- Look for the highest power of (x) in the function.
- If x^n is the highest term then:

$$limit\ f(x) = \frac{coefficient\ of\ x^n\ in\ numerator}{coefficient\ of\ x^n\ in\ denominator}$$

Note : consider the sign in front of the coefficient

Example 1

$Evaluate\ \lim_{x\to\infty} \frac{2-x^2+\ x^4}{2x^4-5x}$ *(highest power of x is 4)*

$$= \frac{1x^4}{2x^4}$$

$$= \mathbf{\frac{1}{2}}$$ *(coefficent of x^4)*

Example 2

$$\lim_{x\to\infty} \quad 6 - \frac{3}{x+1}$$

Cross multiply

$$= \lim_{x\to\infty} \frac{6}{1} - \frac{3}{x+1}$$

$$= \lim_{x\to\infty} \frac{6(x+1)}{1} - \frac{3(1)}{x+1}$$

$$= \frac{6x+6-3}{x+1}$$

$$= \frac{6x+3}{x+1}$$ *(highest power of x is 1)*

$$= \frac{6x}{x}$$

$$= \frac{6}{1}$$ *(coefficient of x)*

$$= \mathbf{6}$$

Student Activity

Evaluate the following limits

(a) $\lim_{x\to\infty} \frac{2x^2-5x^3+3x+4}{-4x+2x^3}$

(b) $\lim_{x\to\infty} \frac{x^2+1}{x^3+1}$

(c) $\lim_{x\to\infty} \quad 4 - \frac{3}{x+1}$

WEEK 3: MONDAY 19/07 TO FRIDAY 23/07

Achievement Indicator:

- To calculate limits.

Type 2: Algebraic Cancellation

Steps

- Take out a common factor
- Factorize numerator and denominator.
- Cancel out the common factor.
- Substitute value of x

Example

Find the limit

$$\lim_{x\to 0}\frac{x^2+x}{x}$$

Solution

Direct substitution

$$\lim_{x\to 0}\frac{x^2+x}{x}$$

$$=\frac{o^2+0}{0}=\frac{0}{0}\therefore\ \ limit\ exists$$

$$\lim_{x\to 0}\frac{x^2+x}{x}$$

$=\lim_{x\to 0}\frac{x(x+1)}{x}$ *(factorise the numerator and cancel x)*

$=\lim_{x\to 0}\ \ x+1$

$=0+1$ *(substitute x-value)*

$=\mathbf{1}$

Type 3: L.Hopital's Rule (differentiation)

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Rules of differentiation

- Bring the power in front.
- Subtract 1 from the power.

$$\boxed{f(x)=x^n\qquad f'(x)=nx^{n-1}}$$

Example

$f(x)$	$f'(x)$
$f(x)=3$	$f'(x)=0$
$f(x)=3x$	$f'(x)=1.3x^{1-1}$ $=3x^0=3(1)=3$
$f(x)$ $=3x^2$	$f'(x)\ =2.3x^{2-1}$ $=6x$
$f(x)=\sqrt{x}$	$f(x)=x^{\frac{1}{2}}$ $f'(x)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}$

Example

Find limit

$$\lim_{x\to -1}\frac{3x^2-3}{x+1}$$

Direct substitution

$$=\frac{3(-1)^2-3}{-1+1}$$

$$=\frac{3-3}{0}$$

$$=\frac{0}{0}\ \therefore\ limit\ exists.$$

Use L.Hopitals* Rule *(differentiate num. and den.)

$$\lim_{x\to -1}\frac{3x^2-3}{x+1}$$

$$=\frac{2.3x^{2-1}-0}{1+0}$$

$$=\frac{6x}{1}$$

$=\frac{6(-1)}{1}$ *(substitute x-value(-1))*

$$=\frac{-6}{1}$$

$$=\mathbf{-6}$$

Student Activity

Evaluate the following limits.

(a) $\lim_{x\to -2}\ \frac{x^2+5x+6}{x^2-4}$

(b) $\lim_{x\to\infty}\ \frac{x^3-6}{7x}$

(c) $\lim_{x\to\infty}\ \frac{4x^2-2x-1}{x^2-2}$

WEEK 4: MONDAY 26/07 TO FRIDAY 30/07

Achievement Indicator:

- To calculate limits.

Limits of Trigonometric Functions

- For the **Indeterminate Form** in trig functions, you probably have to **use** some Trig Identities to compute limits:
  - $\cos^2 x + \sin^2 x = 1$
  - $\sin 2x = 2\sin x.\cos x$
  - $\tan x = \dfrac{\sin x}{\cos x}$
- L' Hôpital's rule where applicable.

  Some derivatives are given below:

$y = \sin x$	$y = \cos x$	$y = \cos bx$	$y = \sin bx$
$y' = \cos x$	$y' = -\sin x$	$y' = -b\sin bx$	$y' = b\cos bx$

Example 1 Evaluate $\lim\limits_{x \to \pi} \dfrac{\sin^2 x + \cos^2 x}{x}$

Answer

Use identity $\cos^2 x + \sin^2 x = 1$

$$\lim_{x \to \pi} \frac{\sin^2 x + \cos^2 x}{x}$$

$$\lim_{x \to \pi} \frac{1}{x}$$

Directly substitute π,

$$\lim_{x \to \pi} \frac{1}{x} = \frac{1}{\pi}$$

Example 2 Evaluate $\lim\limits_{\theta \to \pi} \dfrac{\sin \theta}{\sin 2\theta}$

Answer

Directly substitute π,

$$\lim_{\theta \to \pi} \frac{\sin \theta}{\sin 2\theta} = \frac{0}{0}$$

The limit may exist.

Use identity, cancel then substitute:

$$\lim_{\theta \to \pi} \frac{\sin\theta}{\sin 2\theta}$$

Use identity:
$\sin 2x = 2\sin x.\cos x$

$$\lim_{\theta \to \pi} \frac{\cancel{\sin\theta}}{2\cancel{\sin\theta}\cos\theta}$$

$$\lim_{\theta \to \pi} \frac{1}{2\cos\theta}$$

$$\therefore \lim_{\theta \to \pi} \frac{1}{2\cos\pi}$$

$$= -\frac{1}{2}$$

Student Activity

Find the following limits using trigonometric identities.

1. $\lim\limits_{\theta \to \frac{\pi}{2}} \dfrac{Cos\ \theta}{Sin\ 2\theta}$

2. $\lim\limits_{x \to \pi} \dfrac{Sin\ 2x}{tanx}$

WEEK 5: MONDAY 02/08 TO FRIDAY 06/08

Achievement Indicator:

- To calculate limits using different techniques

Student Activity

1. Evaluate the following limits using the appropriate method.

(a) $\lim_{x \to -7} \frac{x^2-49}{x+7}$

(b) $\lim_{x \to 1} 2x + 4$

(c) $\lim_{x \to \infty} \frac{-5x+2}{2x-3}$

(d) $\lim_{x \to \infty} \frac{7x+3}{9-x}$

(e) $\lim_{x \to 0} \frac{Sin\, x}{2x}$

2. Find ***b*** if $\mathbf{\lim_{x \to 1} \frac{2x^2-bx}{x+3x^2} = 2}$