SUVA SANGAM COLLEGE
YEAR 13
PHYSICS
WEEK 1: MONDAY 05/07 TO FRIDAY 09/07

STRAND	P13.2 Oscillatory Motion
SUB-STRAND	P13.2.1 Simple Harmonic Motion
CONTENT LEARNING OUTCOME	P13.2.1.1 Involve knowledge and understanding of phenomena, concepts, principles and relationships related to SHM
REFERENCE FROM TEXTBOOK	Pg 50-51

Achievement Indicators

- Define and demonstrate understanding of Simple Harmonic Motion
- Describe the characteristics of SHM
- State /calculate where maximum/ minimum values of linear quantities occur in SHM

Simple Harmonic Motion

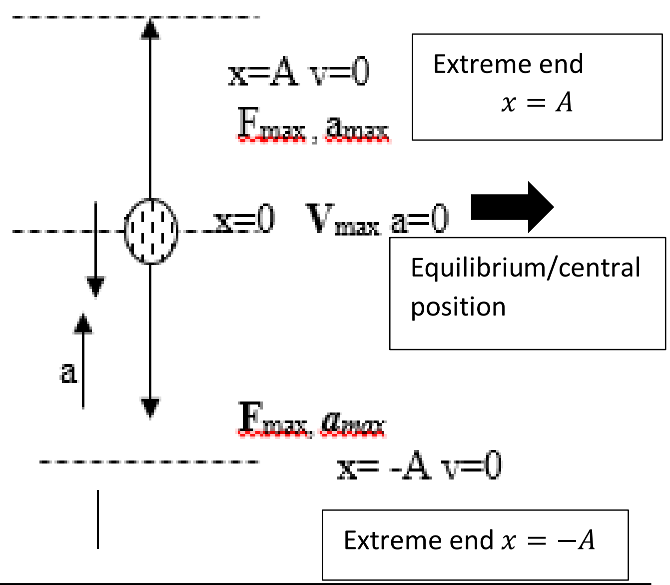
Oscillatory Motion

- Any motion, which repeats over in regular cycles, is called a periodic
- The back and forth movement is called oscillation.
- periodic motion is called simple harmonic motion

Example: Pendulum and Mass on spring

Characteristics of SHM:

1	The acceleration (a) of the object is always directly proportional to the displacement (y) from the equilibrium position.
2	Acceleration and the force is always directed towards the equilibrium position. $(x = 0)$
3	The mass undergoing SHM oscillates between 2 extreme positions on either side of the central point.
4	The displacement of the mass will be taken from the central point, the position the mass will take if allowed to come to rest.
5	The oscillating mass takes exactly the same time to complete one cycle, this time being the period (T) of SHM.
6	The oscillating mass is fastest when passing through the central point and momentarily at rest at the extreme points of the path.
7	The acceleration is always centrally directed and increases to a maximum towards the extremes and is zero at the centre.



Hooke's Law

$$F = -kx$$

k= spring constant (N/m)

Force is always directed towards the equilibrium position and therefore opposite to the displacement from equilibrium

Newtons second Law

$$-kx = ma$$

$$a = -\frac{k}{m}x$$

$a \;\alpha\; -x$

Any motion caused by a force that is proportional to the negative of the displacement must be simple harmonic.

Example 1

Complete the following:

a. Amplitude (A)is the ______________displacement from the central position.

b. Frequency (f) is the number of oscillations per_____________. $\left(Hz = \frac{1}{s} or\_s^{-1} \right)$

c. Period (T) – time taken for one complete _______________.

Solution

a. maximum
b. second
c. revolution

Example 2

FY13 2018 Q9

Force of simple harmonic motion at centre position is always_____________.

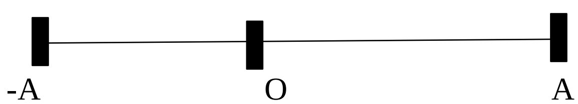
A. zero.
B. negative
C minimum
D. maximum

Solution: A

Example 3

FY13 2017 Q10

A mass is oscillating horizontally in Simple Harmonic Motion about the point O.



Which of the following describes its acceleration and velocity as the mass passes through the point O?

	Acceleration	Velocity
A	zero	Zero
B	maximum	Zero
C	zero	Maximum
D	maximum	maximum

Solution: C

Student Activity

1. **FY13 2019 Q1**

When an object is oscillating with simple harmonic motion, its motion through the equilibrium position can be best described by

A. zero velocity and maximum force

B. zero acceleration and minimum speed

C. zero acceleration and maximum speed

D. zero amplitude and maximum acceleration

2**. FY13 2016 Q9**

The number of oscillations that a particle undergoes per unit time interval is its

A. amplitude

B. frequency

C. angular frequency

D. angular acceleration

3. **FY13 2015 Q10**

In order for an object to move in a simple harmonic motion (SHM) when it is released from its equilibrium position, the object must

A. be suspended like a pendulum.

B. experience a gravitational force.

C. move in a circular path at a constant speed.

D. experience a restoring force proportional to its displacement.

Please check out this video from YouTube explaining the lesson:

1. https://youtu.be/N3JV8WDTBc0

2. https://youtu.be/C-iz3cHLZdA

3. https://youtu.be/NxQQjnpL7U0.

WEEK 2: MONDAY 12/07 TO FRIDAY 16/07

REFERENCE FROM TEXTBOOK	Pg 51-53

Achievement Indicators

-Calculate various quantities from mathematical representation of SHM.

-Derive and solve the problems relating to differential equations of SHM

Displacement of Simple Harmonic Motion

Displacement from equilibrium position

Angular frequency (rad/s)

$\omega t \pm \phi$

Phase of the motion

$$x(t) = A\cos(\omega t \pm \phi)$$

Amplitude (m)

Time (s)

Phase constant/initial phase angle

Angular Frequency

- Measures how rapidly the oscillations are occurring
- The more oscillations per unit time, the higher the value of ω

Formula

In terms of T	In terms of f	In terms of spring constant and mass
$\omega = \dfrac{2\pi}{T}$	$\omega = 2\pi f$	$\omega = \sqrt{\dfrac{k}{m}}$

Spring constant

Displacement ⇨ Velocity ⇨ Acceleration Differentiate	Sin ⇨ Cos Cos ⇨ − Sin Differentiate

Displacement $x(t) = A\cos(\omega t + \phi)$

initial displacement: $t = 0s$

Velocity $v(t) = \dfrac{dx}{dt}$

Differentiate displacement with respect to x

$x = A\cos(\omega t + \phi)$

$\cos(\omega t + \phi)$

$\omega t + \phi$

Differentiate cos

$-\sin(\omega t + \phi)$

Differentiate ω with respect to t

ω

$\dfrac{dx}{dt} = A \cdot -\sin(\omega t + \phi)(\omega)$

$v = -\omega A\sin(\omega t + \phi)$

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$

$v = -\omega A\sin(\omega t + \phi)$

$\sin(\omega t + \phi)$

$\omega t + \phi$

$\cos(\omega t + \phi)$

ω

$\dfrac{dv}{dt} = -\omega A(\omega)(\cos\omega t + \phi)$

$a = -\omega^2 A(\cos\omega t + \phi)$

Velocity	Maximum Velocity
$v = \omega\sqrt{A^2 - x^2}$	$v_{max} = \omega A$
Acceleration	**Maximum Acceleration**
$a = -\omega^2 x$	$a_{max} = \omega^2 A$

Please check out this video from YouTube explaining the lesson:

1. https://youtu.be/f24swwtvTkQ **2. https://youtu.be/y6-NxJz6OEQ**

Example 1: FY13 2015 Q7a

An equation of simple harmonic motion is given as
$y = 2\,sin\,(2.3\,t)$

Calculate

(i) amplitude –

A ω

$y = 2\,sin\,(2.3\,t)$

displacement

A= $Amplitude = 2\,m$

(ii) frequency

$$f = ?$$
$$\omega = 2\pi f$$
$$f = \frac{\omega}{2\pi} = \frac{2.3}{2\pi} = 0.37\ Hz$$

(iii) displacement after $\frac{\pi}{2}$ seconds.

$$substitute\ \frac{\pi}{2}\ in\ the\ displacemnet\ formula.$$
$$\pi = 180°$$

$y = 2\sin(\,2.3\ \times \frac{180}{2}) = 2\sin(207) = -0.91\ m$

(iv) maximum velocity

$$v = \ \omega A$$
$$= 2 \times 2.3 = 4.6\ m/s$$

Example 2 :FY13 2015 Q5

An object undergoing simple harmonic motion (SHM) has its displacement, y, at time t seconds given by the equation.

$$y = 5\sin(4t + \frac{\pi}{4})$$

(i) What is the initial phase angle ϕ?

$\frac{\pi}{4} radian$ or $\frac{180}{4} = 45°$

(ii) Calculate the velocity at time $t = 0\ sec.$

$v\ = \omega\sqrt{A^2 - x^2}$

$\omega = 4\frac{rad}{s}, A = 5m$

Calculate x by substituting $t = 0$ in

$y = 5\sin\left(4(0) + \frac{180}{4}\right)$

$= 3.54\ m$

$= 4\sqrt{5^2 - 3.54^2}$

$= \mathbf{14.14}\ \boldsymbol{m/s}$

Example 3: FY13 2019 Q3

A 0.4 kg mass is connected to a spring with a spring constant of 20 N/m and oscillates on a frictionless horizontal surface with amplitude of 5 cm. Calculate the velocity when the mass is at a displacement of 4 cm.

$k = 20\ N/m, m = 0.4\ kg,\ A = 5cm = \frac{5}{100} = 0.05\ m$

Displacement $= x = 4\ cm = 4 \div 100 = 0.04\ m$

$v\ = \omega\sqrt{A^2 - x^2}$

$calculate\ \omega$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{0.4}} = 7.07\ rad/s$$

$v = 7.07 \times \sqrt{0.05^2 - 0.04^2}$ **= 0.21 m/s**

Student Activity

1. **FY13 2016 Q3a**

An object undergoing simple harmonic motion has its displacement, x, at time, t, seconds given by the equation below.

$$x = 5.2\cos(2\pi t + \frac{\pi}{3})$$

(i) Determine the phase constant of the motion.

(ii) Calculate the displacement at time , $t = 1\ second.$

2. **FY13 2014 Q8**

An object undergoing simple harmonic motion has its displacement, x, at time, t, seconds given by the equation

$$x = 0.5\cos(4\pi t + \frac{\pi}{4})$$

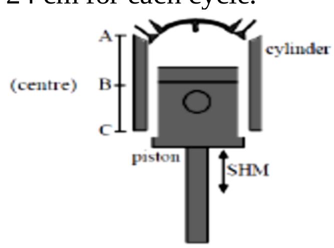
(i) Determine the amplitude and its initial phase angle.

(ii) Calculate, T, the period of the motion.

(iii) Calculate the maximum speed.

3. **FY13 2014 Q2a**

The piston inside a car cylinder oscillates up and down in simple harmonic motion, as shown, at 6000 cycles per minute. It travels up and down through a total distance of 24 cm for each cycle.



(i) What is the amplitude of the piston's motion?

(ii) Calculate the motions angular frequency.

(iii) At what position does the piston have maximum acceleration?

(iv) Calculate the maximum acceleration of the piston.

WEEK 3: MONDAY 19/07 TO FRIDAY 23/07

Achievement Indicators

Relate the gradient to angular velocity	Use the expression for period and angular velocity of a spring and pendulum to solve related quantities

Difference between the Spring and the mass System

	Spring	**Pendulum**
Period (T)	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{l}{g}}$
Frequency (f)	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$
Angular frequency ($\boldsymbol{\omega}$)	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{l}}$
Factors that determines period (dependent)	**Mass** **Spring constant**	**Length of the pendulum** **Acceleration due to gravity**
Factor that does not determine period(independent)	**Amplitude**	**mass**

Example 1

A 1 kg block connected to a light spring with a force constant of 100 N/m is free to oscillate on horizontal frictionless surface. The block is displaced 6 cm from equilibrium position and released from rest.

a) Determine the angular frequency

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} = 10$ rad/s

b) Find the period of its motion

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{100}} = 0.628\ s$$

c) Calculate maximum speed of the block.

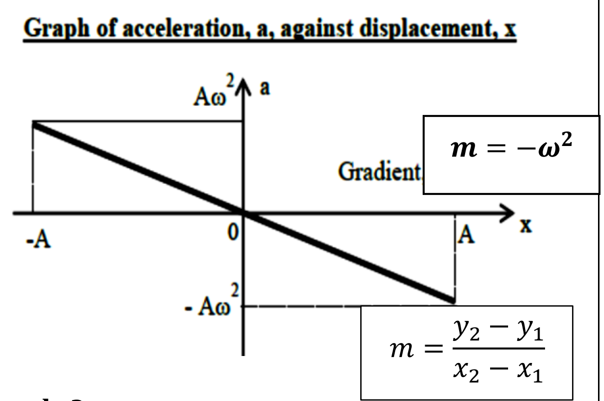
$$v = \omega A$$

$$= 10 \times 0.06 = 0.6\ m/s$$

d) Determine the maximum acceleration

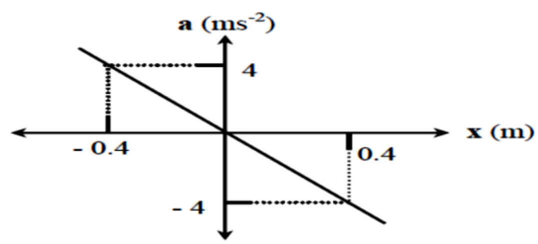
$$a = \omega^2 A = 10^2 \times 0.06 = 6\ m/s^2$$

Graph of acceleration , a, against displacement , x



Example 2

The graph below shows the acceleration against displacement for an object performing SHM.



a) State the relationship between the acceleration and displacement.

Directly proportional to the negative displacement and acceleration is always directed towards the equilibrium position.

$$a\ \alpha -x$$

$$a = -kx$$

b) Calculate the gradient of the graph.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - -4}{-0.4 - 0.4} = -10$$

c) Calculate angular frequency

$$m = -\omega^2$$

$$-10 = -\omega^2$$

$\omega = \sqrt{10} = 3.16\ rad/s$

Please check out this video from YouTube explaining the lesson: https://youtu.be/4CFkXeEDjNE

<u>Student Activity</u>

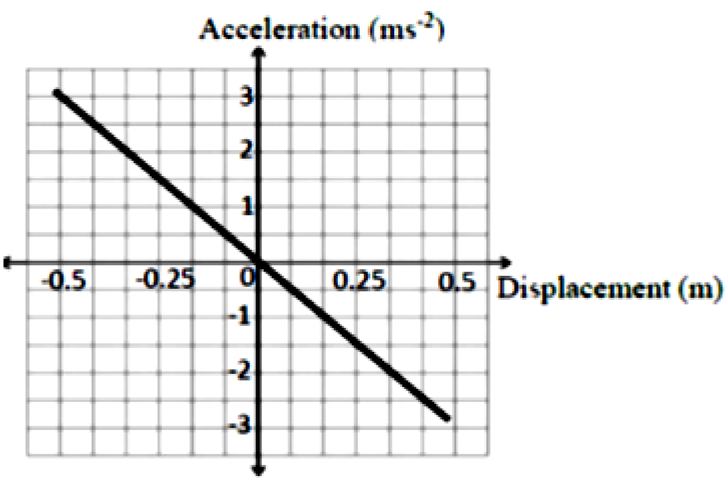
1. A pendulum of length 1.8 m oscillates with amplitude of 0.4 m.

a) What is the period and frequency of the pendulum?

b) Find the velocity at the mid- point of the spring.

c) If the mass of the bob is doubled, calculate the new period.

2. The graph given below shows Acceleration against Displacement for an object performing SHM.



a) Explain clearly how the graph shows that the object is moving with simple harmonic motion.

b) Write an expression of the gradient of the graph in terms of ω.

c) Find the gradient of the graph, and hence, the value of ω.

d) Calculate the period of the SHM.

3**<u>. FY13 2012 Q6a</u>**

A small mass vibrating with SHM has a velocity of 0.5 m/s as it passes through its equilibrium position- the midpoint of the motion

a) What is the velocity of the mass at its maximum displacement?

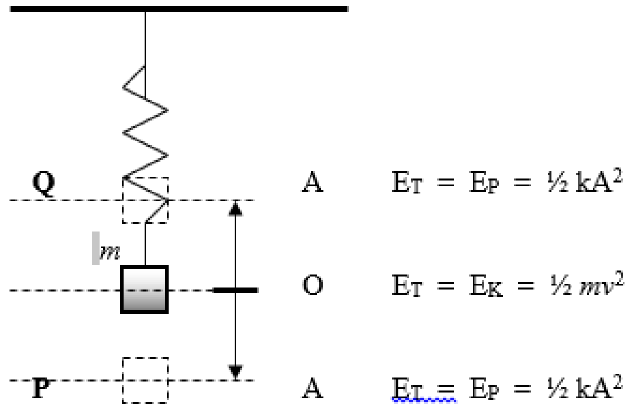
b) If the amplitude of the vibration is 5 cm, what is the period?

WEEK 4: MONDAY 26/07 TO FRIDAY 30/07

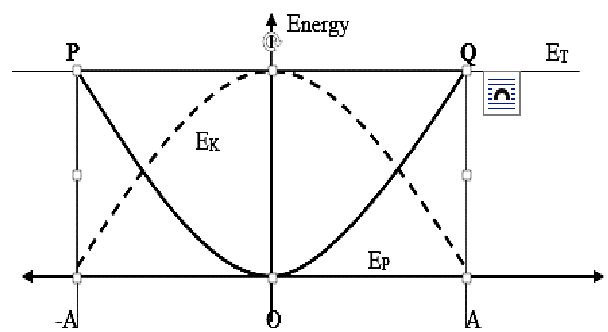
SUB-STRAND	P13.2.2 Energy of SHM
CONTENT LEARNING OUTCOME	P13.2.2.1 Apply the concept of energy in SHM to solve related problems.
REFERENCE FROM TEXTBOOK	Pg 61-63

Achievement Indicators

Explain the concept of energy in SHM	Use conservation of energy ideas to solve problems.	Sketch graph of energies versus displacement

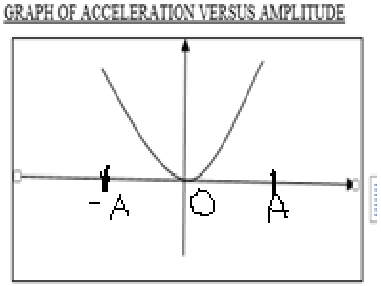


Graph of Energy versus Displacement

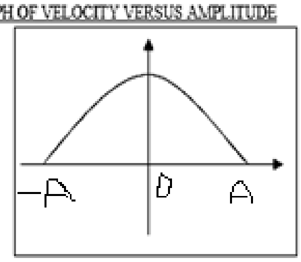


	$x = A$ or $-A$	$x = 0$	Between **0 and A**
Elastic Potential energy	$\frac{1}{2}kA^2$	0	$\frac{1}{2}kA^2$
Kinetic energy	0	$\frac{1}{2}mv^2$ OR $v = \omega A$ $\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}mv^2$ OR $v = \omega A$ $\frac{1}{2}m\omega^2A^2$
Total Energy	$E_T = E_p + E_k$ $E_T = E_p + 0$ $E_T = E_p$ $E_T = E_p = \frac{1}{2}kA^2$	$E_T = E_p + E_k$ $E_T = 0 + E_k$ $E_T = E_k$ $E_T = E_k = \frac{1}{2}m\omega^2A^2$	$E_T = E_p + E_k$ $E_T = \frac{1}{2}kA^2 + \frac{1}{2}m\omega^2A^2$

GRAPH OF ACCELERATION VERSUS AMPLITUDE



GRAPH OF VELOCITY VERSUS AMPLITUDE



Please check out this video from YouTube explaining the lesson:
1. https://youtu.be/TVJ2-knbDD4 2. https://youtu.be/XjkUcJkGd3Y

Example 1: FY13 2011 Q5

A pendulum of length 2 m and a mass of 5 kg swings with amplitude of 0.3 m. Calculate the

a) period (T)

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{2}{9.8}} = 2.84\ s$$

b) total energy of the SHM

$$E_T = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2$$

$$= \frac{1}{2}(5)(2.21)^2(0.3)^2$$

$$= 1.10\text{ J}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.84} = 2.21\ rad/s$$

c) potential energy at the point where the mass is moving with a speed of 0.5 m/s.

$E_T = 1.10$ J

Velocity is given so means kinetic energy

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(5)(0.5)^2 = 0.63\text{ J}$$

$E_p = ?$

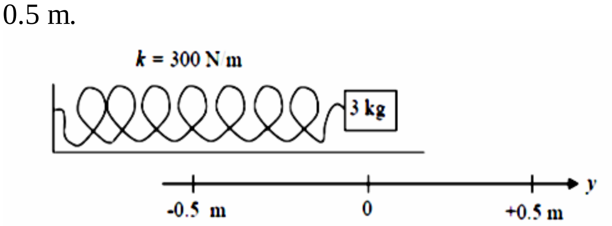
$E_T = E_k + E_p$

$1.10 = 0.63 + E_p$

$E_p = 1.10 - 0.63 = 0.47$ J

Example 2:FY13 2010 Q5

A 3kg mass is attached to a spring ($k = 300\frac{N}{m}$) oscillates on a frictionless surface with an amplitude of 0.5 m.



Calculate the:

a) total energy of the oscillation

$$m = 3\ kg, k = 300\frac{N}{m}, A = 0.5\ m$$

$$E_T = \frac{1}{2}kA^2 = \frac{1}{2}(300)(0.5^2) = 37.5J$$

b) speed of the mass at the equilibrium position.

At equilibrium velocity $= maximum\ \ A = 0$

$$\boldsymbol{E_T = E_p + E_k}$$

$$\boldsymbol{E_T = 0 + E_k}$$

$$\boldsymbol{E_T = E_k}$$

$$E_T = E_k = \frac{1}{2}\boldsymbol{mv^2}$$

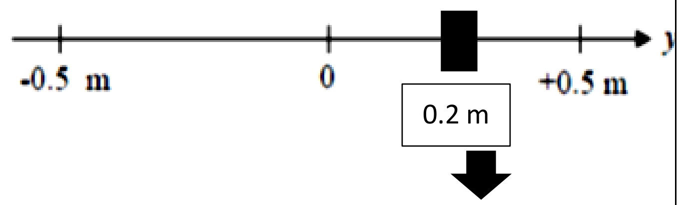
$$37.5 = \frac{1}{2}(3)v^2$$

$$37.5 = 1.5v^2$$

$$v^2 = 25$$

$$v = \sqrt{25} = 5\ m/s$$

c) kinetic and potential energy, 0.2 m from the equilibrium position



$E_k = ?\ E_p = ?$

A= $0.2\ m - calculate\ potential\ energy\ first$

$$E_p = \frac{1}{2}kA^2 = \frac{1}{2}(300)(0.2^2) = 6\text{ J}$$

At A= $0.2\ m$ – there is both kinetic and potential energy

$\boldsymbol{E_T = E_p + E_k}$

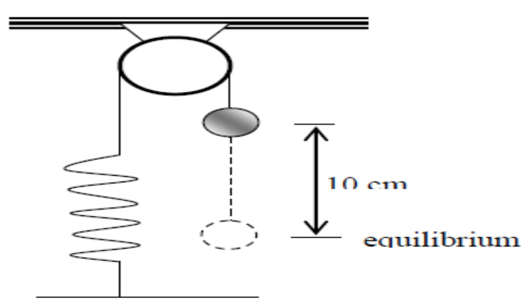
$37.5 = 6 + E_k$

$E_k = 37.5 - 6 = 31.5$ J

Student Activity

1. FY13 2015 Q3c

A 3 kg mass is fastened to a light spring that passes over a pulley. The pulley is frictionless and its inertia can be neglected.



The mass is released from rest when the spring is not stretched. The mass drops 10 cm before coming to rest at the equilibrium. The mass then vibrates with an amplitude of 5 cm after it is pulled and released.

(i) Find the force constant of the spring

(ii) Find the speed of vibration of spring.

(iii) Calculate the total energy of the oscillating mass.

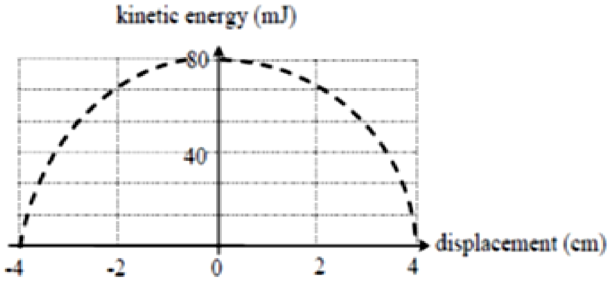
(iv) Calculate the velocity of the mass wen it is 3 cm below the equilibrium

WEEK 5: MONDAY 02/08 TO FRIDAY 06/08

Achievement Indicator : Solve application questions relating to SHM

Example1: FY13 2017 Q2a

The graph given below shows the vibration of kinetic energy with displacement of a particle of mass 0.4 kg performing SHM.



Use the graph to determine the

(i) amplitude of the motion

4 cm or 0.04 m

(ii) total energy of the particle

80 mJ or 80$\times 10^{-3}$ J

(iii) period of the motion

$$m = 0.4\ kg, A = 0.04\ m, E_k = 80 \times 10^{-3}\ J$$

$$T = ?$$

Calculate k

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$E = \frac{1}{2}k\ A^2$$

$$80 \times 10^{-3} = \frac{1}{2}(k)0.04^2$$

$\boldsymbol{k} = \frac{2\times 80\times 10^{-3}}{0.04^2}$= 100 N/ m

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.4}{100}} = 0.4\ s$

Example 2: FY13 2009 Q 7 and 8

Refer to the information given below to answer the questions.

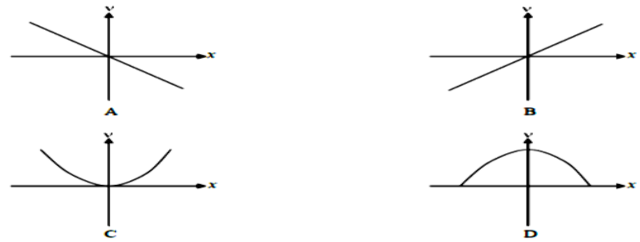
A 3 kg mass attached to a spring with spring constant 300 N/m, undergoes SHM with amplitude 0.4 m.

I. Calculate the total mechanical energy of the simple harmonic oscillator.

$$m = 3\ kg, k = 300\frac{N}{m}, A = 0.4\ m$$

$E_T = \frac{1}{2}kA^2 = \frac{1}{2}\ (300)(0.4^2) = 24$J

II. The graph of the speed of the mass as a function of position is

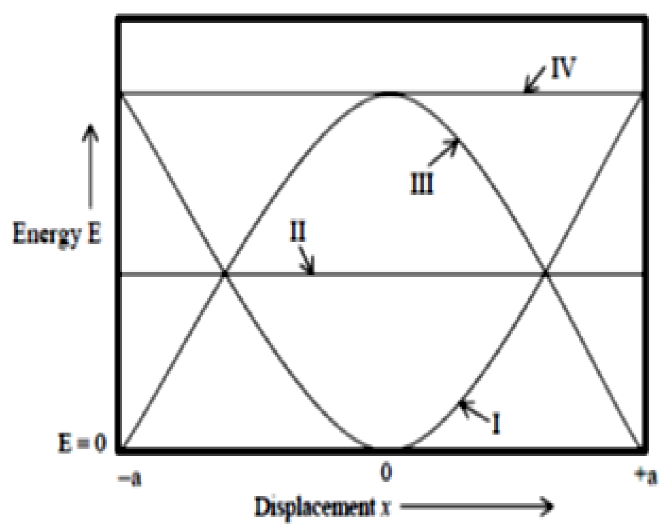


Answer: D

EXAMPLE 3:FY13 2013 Q14 AND 15

Use the graph given below to answers.

Shown below are four graphs of possible vibrations of Energy, E against displacement, x for a system vibrating with simple harmonic motion. It could be associated with a mass attached to the end of a helical spring.



Which graph best shows

a) the kinetic energy of the system varies with the displacement, x?

III

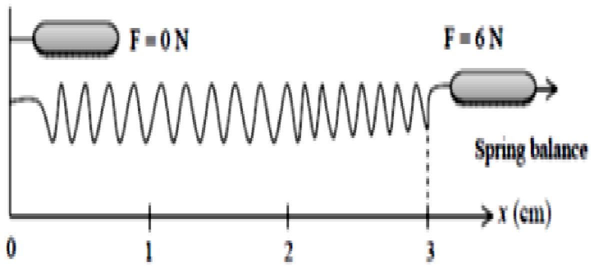
b) potential energy of the system varies with the displacement, x?

I

Student Activity

1. FY13 2013 Q4a

The diagram below shows a 6 N force exerted on a spring.



(i) Calculate the force constant of the spring

The spring balance is removed and is replaced by a 0.5 kg mass. The spring is then pulled a distance of 2 cm, releases and is observed to oscillate in SHM.

(ii) Determine the angular frequency of this oscillation

(iii) Calculate the frequency of the oscillation.

Compute the maximum acceleration of the spring.

(iv) Calculate the total energy of the spring.

2. **FY13 2016 Q5a**

A spring having a force constant of 240 N/m is loaded with a mass of 10 g.

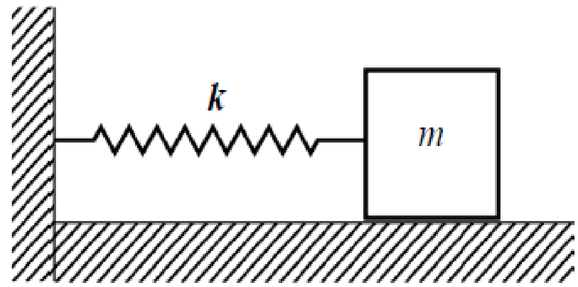
(i) Find the period of vibration.

(ii) If the mass is displaced 30 cm and then released, find the maximum velocity with which it passes through the equilibrium position.

(iii) Calculate the total energy of the vibrating mass.

3. FY13 2018 Q2a

A 5 kg mass is oscillating at the end of a spring with an amplitude of 12 cm. The spring constant, k, 700 N/m.



Calculate the

(i) angular frequency

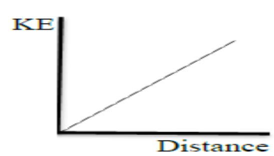
(ii) maximum velocity of the oscillating mass

(iii) maximum acceleration

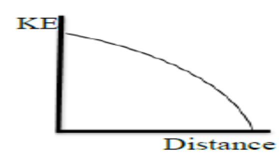
(iv) kinetic energy of the mass at the centre/ equilibrium.

4. **FY13 2015 Q11**

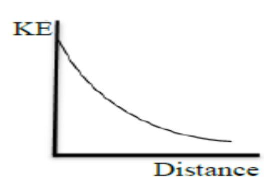
A body executes simple harmonic motion. Which one of the graphs, A to D, best shows the relationship between the kinetic energy of the body and its distance from the centre of oscillation?



A.



B.



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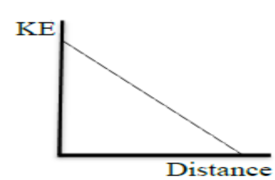

D.