Sangam SKM College - Nadi

Lesson Notes - Week 1

Year 12

Mathematics

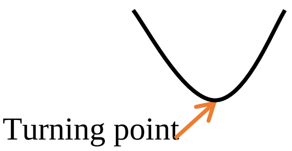
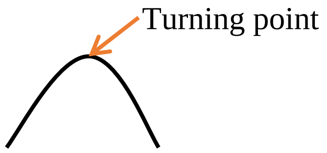
Strand : Algebra

Substrand : Graphs

Content Learning Outcome : Study and interpret Quadratic Graph.

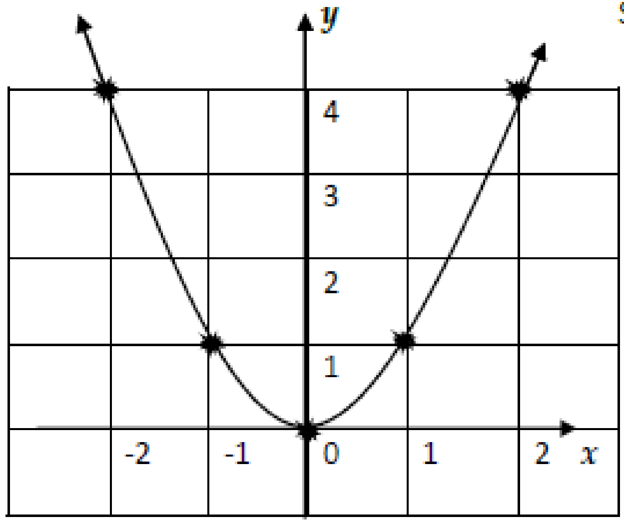
<u>Quadratic Graph</u>

- Quadratic graph will have degree 2, i.e the highest power of 2 eg y = x^2
- The graph will be symmetrical about the turning point (vertex)

POSITIVE SHAPE (y $= ax^2$)	NEGATIVE SHAPE (y $= -ax^2$)
	

- To sketch the graph, we can use :
  1. Table method
  2. Intercept Method - for *x*-intercept let y = 0 , for y - intercept let x= 0 and solve

EXAMPLE 1: Sketch graph of y = x^2
Using tables



Take some x – values, i.e, positive and negative numbers.
Substitute those x values to find y

x	$y = x^2$	(x,y)
-2	$(-2)^2 = 4$	(-2,4)
-1	$(-1)^2 = 1$	(-1,1)
0	$(0)^2 = 0$	(0,0)
1	$(1)^2 = 1$	(1,1)
2	$(-2)^2 = 4$	(2,4)

Example 2   Sketch the graph of $y = x^2 + x - 2$

Using intercept method

x - intercept (y = 0)

$0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$ (Type 1 factorisation)

$0 = x + 2$ $0 = x - 1$

$\boldsymbol{x = -2}$ $\boldsymbol{x = 1}$

y - intercept (x = 0)

$y = x^2 + x - 2$

$y = (0)^2 + 0 - 2$

$\boldsymbol{y = -2}$

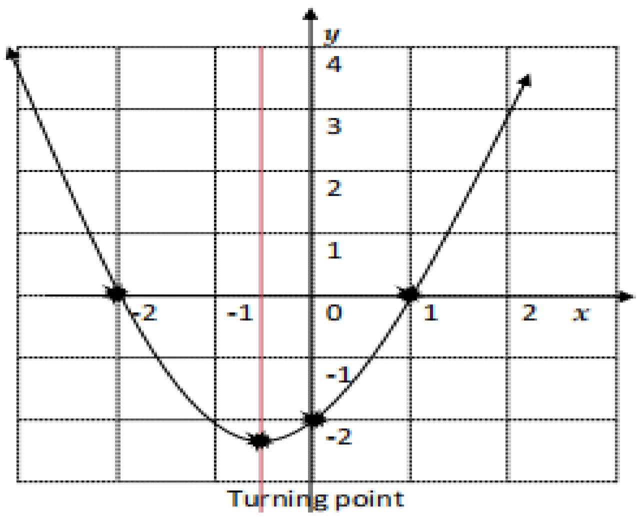
Turning Point (Vertex)

$x = \frac{x_1+x_2}{2} = \frac{-2+1}{2} = \frac{-1}{2}$

$y = x^2 + x - 2$

$y = (-0.5)^2 + (-0.5) - 2$

$y = -2.25$



Activity

Sketch the following graphs:

1. y = x^2 + 3x + 2

2. $f(x) = 3x - x^2$

3. $y = (2x - 1)(x - 3)$

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Lesson Notes - Week 2

Year 12

Mathematics

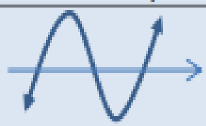
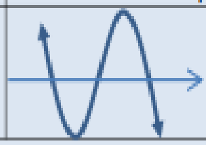
Strand : Algebra

Sub-strand : Graphs

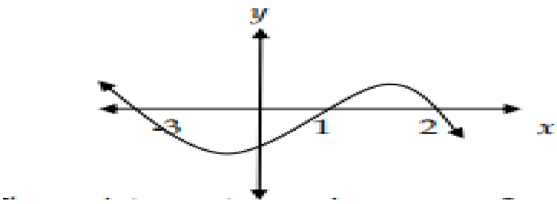
Content Learning Outcome : Study and interpret Cubic Graph.

Cubic Function

- Cubic equation has highest power of 3 , e.g $y = x^3$
- To sketch a cubic graph, the intercept method can be used.

GRAPH	POSITIVE SHAPE $y = +ax^3$	NEGATIVE SHAPE $y = -ax^3$
Cubic function / graph		

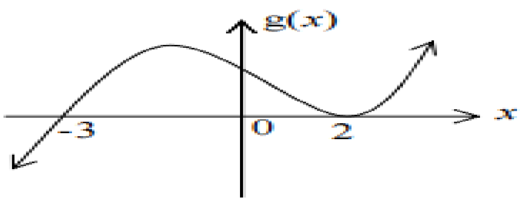
***Example 1*:** Find the equation of the graph shown below:



The x-intercepts are $x = -3$, $x = 1$ and $x = 2$. Hence take it to the left hand side with the x. And the shape of the graph is negative so put a negative sign. Thus the equation will be $\mathbf{y = -(x+3)(x-1)(x-2)}$

Example 2

Give the equation of the cubic graph shown below



The x-intercepts are $x = -3$ and $x = 2$. And the shape of the graph is positive.

Thus the equation of the graph will be $g(x) = (x - 2)^2(x + 3)$. The repeated factor $(x - 2)^2$ means the graph turns at $x = 2$

<u>Example 3</u>

Sketch the graph of $y = (x + 2)^2(1 - x)$, showing all the intercepts

<u>x -intercept($y = 0$)</u>

$y = (x + 2)^2(1 - x)$

$0 = (x + 2)^2(1 - x)$

$(x + 2)^2 = 0 \qquad (1 - x) = 0$

$\boldsymbol{x = -2} \qquad \boldsymbol{x = 1}$

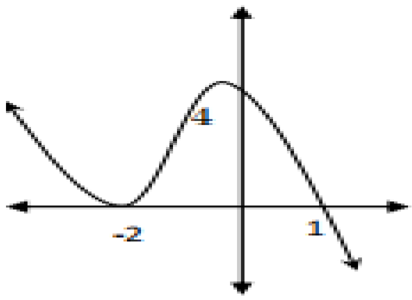
<u>y -intercept($x = 0$)</u>

$y = (x + 2)^2(1 - x)$

$y = (0 + 2)^2(1 - 0)$

$\boldsymbol{y = 4}$

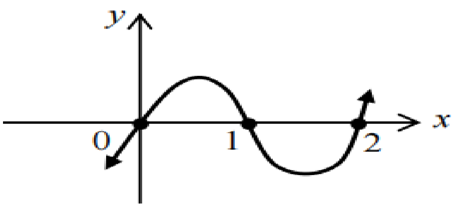
Note: The repeated factor in $y = (x + 2)^2(1 - x)$ is $(x + 2)^2$, that means the graph turns at $x = -2$ as shown below.



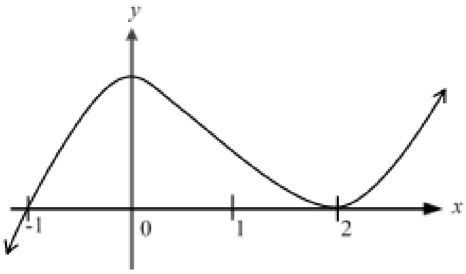
<u>Activity</u>

Give the equation of the cubic functions shown below:

1.



2.



3. Sketch the graph of the cubic function given by the equation $y = x^2(x + 3)$

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Lesson Notes - Week 3

Year 12

Mathematics

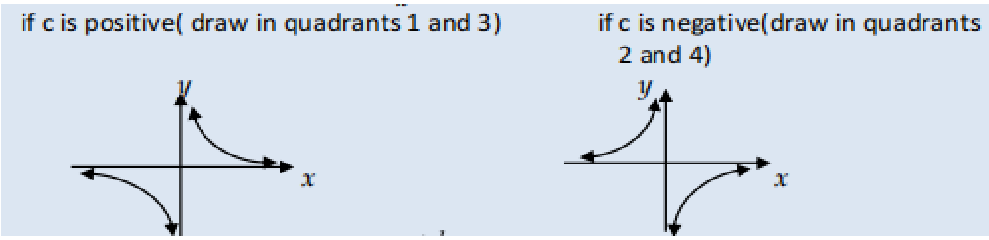
Strand : Algebra

Sub-strand : Graphs

Content Learning Outcome : Study and interpret Hyperbolic Function.

<u>Hyperbolic Graph</u>

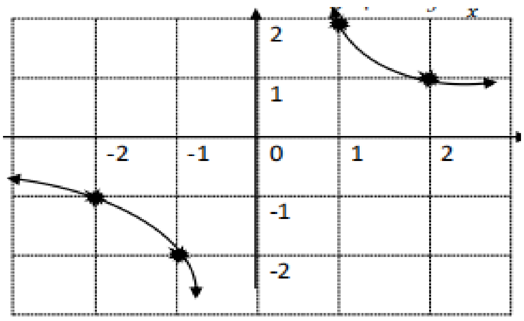
- Hyperbola will have a basic form of $\boldsymbol{x\, y = c}$, where c is a constant. Making y the subject yields: $\boldsymbol{y = \frac{c}{x}}$. Table of values method can be used to sketch the graph.



- The **rectangular form** of a hyperbola is given by: $\boldsymbol{y = \frac{ax + b}{cx + d}}$
- ***<u>Steps to sketch</u>:***
  1. *x - intercept* - let numerator = 0 and solve , i.e $ax + b = 0$
  2. *y - intercept* - let $x = 0$ and solve
  3. *Vertical asymptote* - let denominator = 0 and solve, i.e $cx + d = 0$
  4. *Horizontal asymptote* - divide the coefficients of the variable x , i.e $y = \frac{a}{c}$
- **Asymptotes** are dotted lines that the graph will never cross.

<u>Example 1</u> : Sketch the graph of $y = \frac{2}{x}$

x	$y = \frac{2}{x}$	(x,y)
-2	$\frac{2}{-2} = -1$	(-2,-1)
-1	$\frac{2}{-1} = -2$	(-1,-2)
0	$\frac{2}{0} = undefined$	-
1	$\frac{2}{1} = 2$	(1,2)
2	$\frac{2}{2} = 1$	(2,1)



Example 2 **:** A function is given as $g(x) = 2 - \frac{3x}{x-1}$

(i) Express $g(x)$ in the form $y = \frac{ax+b}{cx+d}$

Make denominator the same

$$g(x) = 2 - \frac{3x}{x-1}$$

$$= \frac{2}{1} - \frac{3x}{x-1}$$

Cross multiply

$$= \frac{2(x-1) - 3x(1)}{(x-1)}$$

Distributive law

Why?

and simplify

$$= \frac{2x - 2 - 3x}{(x-1)}$$

$$\therefore g(x) = \frac{-x-2}{x-1}$$

(ii) Find the x and y intercepts

<u>x-intercept (numerator = 0)</u>

$-x - 2 = 0$

$\boldsymbol{x = -2}$

<u>y-intercept (x = 0)</u>

$g(x) = \frac{-x-2}{x-1}$

$= \frac{-0-2}{0-1}$

$\boldsymbol{y = 2}$

(iii) State the equation of the vertical and horizontal asymptotes

<u>Vertical asymptote (denominator = 0)</u>

$x - 1 = 0$

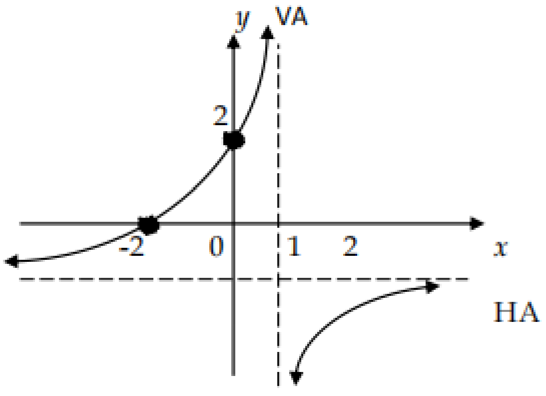
$\boldsymbol{x = 1}$

<u>Horizontal Asympyote</u>

$y = \frac{a}{c}$

$y = \frac{-1}{1}$ $\quad$ $\boldsymbol{y = -1}$

(iv) Hence sketch the graph of $g(x)$



<u>Activity</u>

Sketch the given hyperbolic functions :

1. $y = \frac{x+1}{x-2}$ $\qquad$ 2. $y = \frac{x-3}{x+1}$ $\qquad$ 3. $y = \frac{4}{x-2} + 3$