Sangam SKM College – Nadi

Lesson Notes: Week 1

Year 13

Mathematics

Strand: Algebra

Sub Strand: Sequences

Content Learning Outcome: Study partial sum, convergence and divergence of sequences**.**

SEQUENCES

- Sequence is an ordered list of numbers for e.g. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$
- Series is obtained by adding terms in a sequence for e.g. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$
- Partial sum, S_n is the sum of a finite number of consecutive terms beginning with the first term.
  $S_1 = T_1$ $\quad (T_1 \textit{ is the first term})$
  $S_2 = T_1 + T_2$
  $S_3 = T_1 + T_2 + T_3$

- Sequences which approach a definite value are said to converge.
  If a sequence has a limit, we say the sequence is convergent and the sequence converges to a limit. Otherwise, the sequence is divergent.

Example 1:

A sequence is defined by $T_n = 3n$

a) Find the first four terms of this sequence.

$T_n = 3n$

$T_1 = 3(1) = 3$

$T_2 = 3(2) = 6$

$T_3 = 3(3) = 9$

$T_4 = 3(4) = 12$

The terms of the sequence are

<3, 6, 9, 12, … >

b) Write as partial sum.

$S_1 = 3$

$S_2 = 3 + 6 = 9$

$S_3 = 9 + 9 = 18$

$S_4 = 18 + 12 = 30$

The sequence of partial sum are

<3, 9, 18, 30, … >

Example 2:

Determine whether the sequence $a_n = \frac{6n+3}{n-8}$ converge or diverge, and if it converges, give the value to which it converges to.

$\lim_{n \to \infty} \frac{6n+3}{n-8}$ = *6 (using L'Hopital's Rule)*

Thus, it is a converging sequence and it converges to 6.

Example 3:

A sequence <a_n> is defined by a_n = 2n + 1

a) List the first four terms
The first four terms of the sequence 2n + 1 = < 3, 5, 7, 9, … >

b) What is the limit of this sequence?
$= \lim_{n \to \infty} 2n + 1$
$= 2\infty + 1$
$= \infty$

c) Determine whether the sequence converge or diverge.
The terms of the sequence increase without bounds (goes to infinity) so the sequence diverges.
Note: If the limit is infinity, the sequence diverges.

Activity: Year 13 Mathematics textbook page 131: Exercise 6.1.1 - Questions 1, 3, 5

1. A sequence <a_n> is defined by
$a_n = \frac{7n+3}{n-9}$
   a) Find the first four terms of the sequence.
   b) Find the first three terms of the sequence of partial sums.
   c) Find $\lim_{n \to \infty} \frac{7n+3}{n-9}$
   d) Explain why the sequence converges.

3. A sequence <a_n> is defined by
$a_n = \frac{n+2}{n^2}$
   a) Find the first two terms of the sequence of partial sums.
   b) Determine whether a sequence converges or diverges, and if it converges, give the value to which it converges to.

5. A sequence <a_n> is defined by a_n = 3n + 4
Does it converge or diverge. Explain

Sangam SKM College – Nadi

Lesson Notes: Week 2

Year 13

Mathematics

Strand: Algebra

Sub Strand: Binomial Theorem

Content Learning Outcome: Use Binomial Theorem to expand expressions of the form $(x + \mathbf{a})^{\mathbf{n}}$.

BINOMIAL THEOREM

- FACTORIAL ! - The factorial of a number (symbol n!) is defined as:

  n! = n x (n - 1) x (n- 2) x (n -3) x … x 3 x 2 x 1, n ϵ N

  Note: n! is read as "n factorial"

  e.g. 5! =5 x 4 x 3 x 2 x 1 = 120

  (This can be directly found using the calculator which has the key ! Press 5! =)

- COMBINATIONS n_{c_r} or $\binom{n}{r}$ - a combination is a selection of a certain number of elements from a set where the order of elements is not taken into account. The number of possible combinations of "r" elements from "n" things is denoted by:

  $$\binom{n}{r} = \frac{n!}{(n-r)!\ r!}$$

  $$\binom{10}{6} = \frac{10!}{(10-6)!\,.\ 6!} = 210$$

  (This can also be found using the n_{c_r} key on the calculator Press 10 n_{c_r} 6 =)

- BINOMIAL THEOREM

  The binomial theorem provides a useful method for raising any binomial to a nonnegative integral power:

  $$(x+\mathbf{a})^{\mathbf{n}} = \binom{\mathbf{n}}{\mathbf{0}} x^{\mathbf{n-0}}\,\mathbf{a}^{\mathbf{0}} + \binom{\mathbf{n}}{\mathbf{1}} x^{\mathbf{n-1}}\,\mathbf{a}^{\mathbf{1}} + \binom{\mathbf{n}}{\mathbf{2}} x^{\mathbf{n-2}}\,\mathbf{a}^{\mathbf{2}} + \ldots + \binom{\mathbf{n}}{\mathbf{n}} x^{\mathbf{n-n}}\,\mathbf{a}^{\mathbf{n}}$$

Example 1: Expand $(x+a)^3$ using the binomial theorem.

$$(x+a)^3 = \binom{3}{0} x^{3-0}\, a^0 + \binom{3}{1} x^{3-1}\, a^1 + \binom{3}{2} x^{3-2}\, a^2 + \binom{3}{3} x^{3-3}\, a^3$$

$$= 1.\, x^3.\, 1 \;+\; 3.\, x^2\,.\, a \;+\; 3.\, x^1\,.\, a^2 \;+\; 1.\, 1.\, a^3$$

$$= x^3 + 3x^2 a + 3x\, a^2 + a^3$$

Note: the power of x decreases from n to 0 while the power of a increases from 0 to n.

Example 2: Expand $(2x - \frac{1}{x^2})^3$ using the binomial theorem

$$(2x - \tfrac{1}{x^2})^3 = \binom{3}{0} (2x)^3 (-\tfrac{1}{x^2})^0 + \binom{3}{1} (2x)^2 (-\tfrac{1}{x^2})^1 + \binom{3}{2} (2x)^1 (-\tfrac{1}{x^2})^2 + \binom{3}{3} (2x)^0 (-\tfrac{1}{x^2})^3$$

$$= 1.\, 8x^3.\, 1 \;+\; 3.\, 4x^2.\, (-\frac{1}{x^2}) \;+\; 3.\, 2x.\, (\frac{1}{x^4}) \;+\; 1.\, 1.\, (-\frac{1}{x^6})$$

$$= 8x^3 - 12 + \frac{6}{x^3} - \frac{1}{x^6}$$

Example 3: Write down the first four terms in the expansion of $(2x + y^2)^9$

$$(2x + y^2)^9 = \binom{9}{0} (2x)^9 (y^2)^0 + \binom{9}{1} (2x)^8 (y^2)^1 + \binom{9}{2} (2x)^7 (y^2)^2 + \binom{9}{3} (2x)^6 (y^2)^3 + \ldots$$

$$= 1.(2x)^9.1 + 9.(2x)^8.y^2 + 36.(2x)^7.y^4 + 84.(2x)^6.y^6 + \ldots$$

$$= 512\, x^9 + 2304\, x^8\, y^2 + 4608\, x^7\, y^4 + 5376\, x^6\, y^6 + \ldots$$

Activity:

1. Use the binomial theorem to expand and simplify $(x + y)^4$
2. Expand and simplify $(2x - \frac{1}{2})^4$
3. Use the binomial theorem to find the first three terms of $\left(2x - \frac{1}{x}\right)^5$

Sangam SKM College – Nadi

Lesson Notes: Week 3

Year 13

Mathematics

Strand: Algebra

Sub Strand: Binomial Theorem

Content Learning Outcome: Use general term formula to find particular term.

General Term Formula: $T_{r+1} = \binom{n}{r} x^{n-r} a^r$

The $(r+1)^{th}$ term which is the general term is given by $T_{r+1} = \binom{n}{r} x^{n-r} a^r$ and $\binom{n}{r}$ is the binomial coefficient.

Example**:** A term in an expansion of $(a + b)^n$ is $\binom{p}{k} (x^2)^3 (-\frac{2}{x})^5$.

Find a, b, n, p and k.

$a = x^2$ $b = -\frac{2}{x}$ $k = 5$ $n = p = 8$

Finding a Particular Term

Example 1:

Find the fourth term in the expansion of $(x - \frac{2}{x})^8$

$T_{r+1} = \binom{n}{r} x^{n-r} a^r$

For the fourth term, r = 3

$T_{(3+1)} = \binom{8}{3} x^{8-3} (-\frac{2}{x})^3$

$= 56. \ x^5 \cdot \frac{-8}{x^3}$

$T_4 = -448\, x^2$

Example 2:

What is the third term of $(a-3)^{10}$?

$T_{r+1} = \binom{n}{r} x^{n-r} a^r$

For the third term, r = 2

$T_{(2+1)} = \binom{10}{2} a^{10-2} (-3)^2$

$= 45. \ a^8 \cdot 9$

$T_3 = 405\, a^8$

Finding the Coefficient

Example 1:
Find the coefficient of x^4 in the expansion of $(x + 1)^{10}$

$T_{(r+1)} = \binom{10}{r} x^{10-r} (1)^r$

To solve for r: $x^{10-r} = x^4$
$10 - r = 4$
$r = 6$

replace r with 6

$= \binom{10}{6} x^{10-6} (1)^6$

$= 210 . x^4 . 1$

$= 210x^4$

$\therefore$ coefficient is 210

Example 2:
Find the coefficient of x^3y^4 in the expansion of $(x + 2y)^7$

$T_{(r+1)} = \binom{7}{r} x^{7-r} (2y)^r$

To solve for r: $x^{7-r} = x^3$
$7 - r = 3$
$r = 4$

replace r with 4

$= \binom{7}{4} x^{7-4} (2y)^4$

$= 35. x^3 . 16y^4$

$= 560 x^3 y^4$

$\therefore$ coefficient is 560

Finding Constant Term (term independent of *x*)

Example 1: Find the term independent of x in the expansion of $(x - \frac{8}{3})^6$.

$T_{(r+1)} = \binom{6}{r} x^{6-r} (- \frac{8}{3})^r$

To solve for r: $x^{6-r} = x^0$
$6 - r = 0$
$r = 6$

$= \binom{6}{6} x^{6-6} (- \frac{8}{3})^6$

$= 1. 1. \frac{262\,144}{729}$

$= \frac{262144}{729}$

$\therefore$ Constant term is $\frac{262144}{729}$

Example 2: Find the constant term in the expansion of $(x + \frac{1}{x})^8$.

$T_{r+1} = \binom{8}{r} x^{8-r} (\frac{1}{x})^r$

To solve for r: $x^{8-r} . x^{-r} = x^0$
$8 - r - r = 0$
$r = 4$

$= \binom{8}{4} x^{8-4} (\frac{1}{x})^4$

$= 70. x^4. \frac{1}{x^4}$

$= 70$

$\therefore$ Constant term is 70

Activity:

1. Find the fourth term in the expansion of $(5 + x)^6$
2. Find the coefficient of x^4 in the expansion of $(1 - 2x)^9$
3. Find the term independent of x in the expansion of $(3x^2 - \frac{1}{x})^{12}$