



3055 BA SANGAM COLLEGE

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Worksheet 9

School: Ba Sangam College

Year / Level: 13

Subject: Mathematics

Name of student: _____

Strand	4 – Trigonometry
Sub strand	4.3 – Trigonometric Equations
Content Learning Outcome	Choose suitable strategies for finding solutions to Trigonometric Equations.

Solving Trigonometric Equations

Ref. Yr 13 Mathematics Textbook Pg 91 – 95

When solving any trigonometric equation, emphasis must be given to the quadrants.

Quadrant II Sine +	Quadrant I All +
Tangent +	Cosine +
Quadrant III	Quadrant IV

Mnemonic
All Science Teachers Cry
or
Add Sugar To Coffee

- If you look at the quadrants, the designated trig expressions will be positive, the others will be negative. Further simplifying,

Note: for complex equations, you may use the identities.

Example 1

Find the solution set for $2\cos\theta + \sqrt{3} = 0, 0^\circ \leq \theta \leq 360^\circ$

Answer

The angle to be between $0 - 360^\circ$

$$2\cos\theta + \sqrt{3} = 0, \quad 0^\circ \leq \theta \leq 360^\circ$$

Last

$$2\cos\theta + \sqrt{3} - \sqrt{3} = 0 - \sqrt{3}$$

$$\frac{2\cos\theta}{2} = \frac{-\sqrt{3}}{2}$$

$$\cos\theta = \frac{-\sqrt{3}}{2}$$

We reached at the trig expression: Consider the two quadrants. But before that, find the acute angle by ignoring the negative sign (-). Note that calculator Mode to be in degrees.

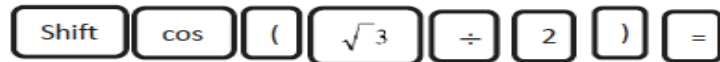
- **Acute angle:**

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

$$\alpha = 30^\circ$$



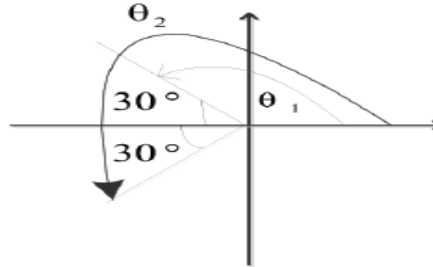
When dealing with surds,
press the division sign (\div), that is Press



- Use quadrants to find the angles θ_1 and θ_2 . Consider negative sign ($-$) of Cos, that is in Q II / III

$$\theta_1 = 180 - 30 = 150^\circ$$

$$\theta_2 = 180 + 30 = 210^\circ$$



$$\theta = 150^\circ, 210^\circ \text{ or } \theta \in \{150^\circ, 210^\circ\}$$

Example 2

Solve the equation $3\sin(x - 45^\circ) = \frac{3\sqrt{3}}{2}$ for $0 \leq x \leq 360^\circ$.

Answer

$$3\sin(x - 45^\circ) = \frac{3\sqrt{3}}{2} \quad 0 \leq x \leq 360^\circ$$

$$\sin(x - 45^\circ) = \frac{\sqrt{3}}{2}$$

$$(x - 45^\circ) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

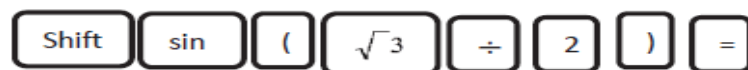
We reached at the trig expression: Consider the two quadrants. But before that, find the acute angle. Note that calculator Mode to be in degrees.

Acute angle:

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$



When dealing with surds,
press the division sign (\div), that is Press



Sine is positive, the angle lies in the 1st and 2nd quadrants of the xy -plane.

$$\theta_1 = 60^\circ$$

$$\theta_2 = 180 - 60 = 120^\circ$$

$$(x - 45^\circ) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$(x - 45^\circ) = \theta_2$$

$$(x - 45^\circ) = 60^\circ$$

$$(x - 45^\circ) = 120^\circ$$

$$x = 60^\circ + 45^\circ = 105^\circ$$

$$x = 120^\circ + 45^\circ = 165^\circ$$

$$\theta = 105^\circ, 165^\circ \text{ or } \theta \in \{105^\circ, 165^\circ\}$$

EXAMPLE 3 Solve $2\cos 2x = \sqrt{3}$ for $0 \leq x \leq 2\pi$

$$2 \cos 2x = \sqrt{3}$$

Last

$$0 \leq x \leq 2\pi \text{ Angle to be between } 0 - 360^\circ$$

$$\frac{2 \cos 2x}{2} = \frac{\sqrt{3}}{2}$$

$$\cos 2x = \frac{\sqrt{3}}{2}$$

We reached at the trig expression and the value in front of the variable is 2, so it has 4 solutions.

If you find easier in degrees, you may do so.

Once all solutions are found, then you can convert to radians as the question requires you to give answers in radians.

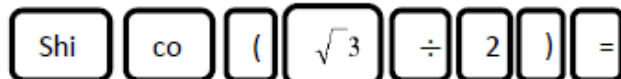
Acute angle:

$$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

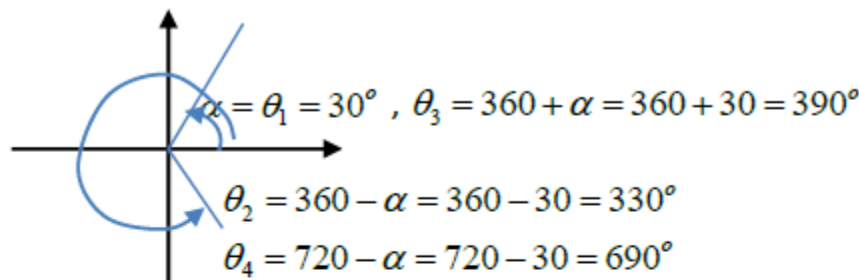


(when dealing with surds, press the division sign (\div), that is Press

$$\alpha = 30^\circ$$



- Use quadrants to find the **four** angles: Cos is positive in Quadrants I / IV



$$2x = \{30^\circ, 330^\circ, 390^\circ, 690^\circ\}$$

Divide by 2, to all the angles

$$\frac{2x}{2} = \left\{ \frac{30^\circ}{2}, \frac{330^\circ}{2}, \frac{390^\circ}{2}, \frac{690^\circ}{2} \right\}$$

$$x = \{15^\circ, 165^\circ, 195^\circ, 345^\circ\}$$

To convert in radians, divide by 180 and multiply by π :

$$x = \left\{ \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12} \right\}$$

ACTIVITY: Solve**(2 marks each)**

1. $2\cos x = 1$, for $0^\circ \leq x \leq 360^\circ$

2. $2 \sin x + \sqrt{3} = 0$, for $0^\circ \leq x \leq 360^\circ$.

3. $\cos 2x = \frac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$.

4. $2 \cos \left(\theta - \frac{\pi}{4} \right) = 1$ for $0 \leq \theta \leq 2\pi$.

5. $\tan 2x - 1 = 0$, for $0^\circ \leq x \leq 360^\circ$

6. $\sin \left(2x + \frac{\pi}{4} \right) = 1$ for $0 \leq x \leq 2\pi$