3055 BA SANGAM COLLEGE

PH: 6674003/9264117 E-mail: basangam@connect.com.fj

WEEK 11

School: Ba Sangam College **Year / Level:** 13
Subject: Mathematics **Name of student:** ______________________

Strand	4 – Trigonometry
Sub strand	4.4 – Application of Addition Formulae
Content Learning Outcome	Transform trigonometric expressions into simple ones

Transformation of trigonometric expressions of the form $ACos\,\theta + BSin\,\theta$

Ref. Yr 13 Mathematics Textbook Pg 96-101

Addition Formulae

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(i) $a\,Cos\,\theta \pm b\,Sin\theta = r\,Cos\,(\theta \pm \alpha)$

(ii) $a\,Cos\,\theta \pm b\,Sin\theta = r\,Sin\,(\theta \pm \alpha)$

where α is an angle to be found and r is the modulus i.e. $r = \sqrt{a^2 + b^2}$ and a and b are ***coefficients*** of $Cos\,\theta$ and $Sin\,\theta$ respectively.

Example : **Express $y = 2\,Cos\theta + 3Sin\theta$ in the form of $R\,Sin\,(\theta + \alpha)$.**

Answer

$a = 2, b = 3 \quad \therefore r = \sqrt{2^2 + 3^2}$
$r = \sqrt{13}$

Apply Addition law: • $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$Sin\,(\theta + \alpha) = [\,Sin\,\theta\,Cos\,\alpha + Cos\,\theta\,Sin\,\alpha]$

$r\,Sin\,(\theta + \alpha) = r\,Sin\,\theta\,Cos\,\alpha + r\,Cos\,\theta\,Sin\,\alpha$

substituting r yields: $\sqrt{13}\,Sin\,\theta\,Cos\,\alpha + \sqrt{13}\,Cos\,\theta\,Sin\,\alpha$

Thus in general form, $2\,Cos\,\theta + 3\,Sin\theta$

$= \sqrt{13}\,Sin\,\alpha\,Cos\,\theta + \sqrt{13}\,Cos\,\alpha\,Sin\,\theta$

Compare Sine & Cosine functions

$2 = \sqrt{13}\,Sin\,\alpha$	$3 = \sqrt{13}\,Cos\,\alpha$
$\sin\alpha = \frac{2}{\sqrt{13}}$	$\cos\alpha = \frac{3}{\sqrt{13}}$
$\alpha = \sin^{-1}\left(\frac{2}{\sqrt{13}}\right)$	$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$
$\alpha = 33.69^o$	$\alpha = 33.69^o$

Since both give the same value for α,

$\therefore \quad 2\,Cos\,\theta + 3\,Sin\,\theta = \sqrt{13}\,Sin\,(\theta + 33.69^o)$

Example 2: Express $y = 2\,Cos\theta + \sqrt{2}\,Sin\theta$ *in the form of* $R\,Cos\,(\theta + \alpha)$.

Answer

$a = 2, b = \sqrt{2}\ \therefore r = \sqrt{\sqrt{2}^2 + 2^2}$

$r = \sqrt{6}$

Apply Addition law: • $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

And substituting r yields: $\sqrt{6}\cos\theta\,Cos\,\alpha - \sqrt{6}\,Sin\,\theta\,Sin\,\alpha$

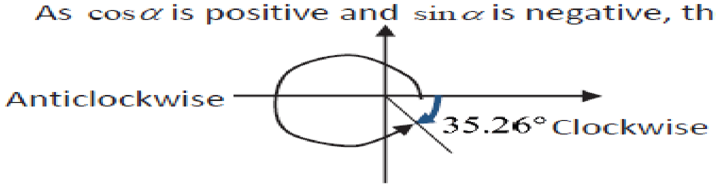
Thus in general form, $2\,Cos\,\theta + \sqrt{2}\,Sin\,\theta$

$= \sqrt{6}\,Cos\,\alpha\,cos\,\theta + \ -\sqrt{6}\,Sin\,\alpha\,Sin\,\theta$

Compare Sine & Cosine functions

$2 = \sqrt{6}\,Cos\,\alpha$	$\sqrt{2} = -\sqrt{6}\,Sin\,\alpha$
$\cos\alpha = \frac{2}{\sqrt{6}}$	$\sin\alpha = \frac{\sqrt{2}}{-\sqrt{6}}$
$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$	$\alpha = \sin^{-1}\left(-\frac{\sqrt{2}}{\sqrt{6}}\right)$
$\alpha = 35.26^o$	$\alpha = -35.26^o$

As $\cos\alpha$ is positive and $\sin\alpha$ is negative, the angle lies in the 4th quadrant



Anticlockwise angle is positive
Clockwise angle is negative

$\alpha = 360 - 35.26$

$= 324.74°\ or -35.26°$

$\therefore \quad 2\ Cos\ \theta + \sqrt{2}\ Sin\theta = \sqrt{6}\ \cos(\theta - 35.26°)$

ACTIVITY: *(4 marks each)*

1. Express $y=2\sin\theta-\cos\theta$ in the form $R\sin(\theta-\alpha)$	2. Express $y=\sqrt{3}\cos\theta-\sin\theta$ in the form $R\sin(\theta-\alpha)$.	3. Express $y=\sqrt{3}\cos\theta+\sin\theta$ in the form $R\cos(\theta+\alpha)$.