3055 BA SANGAM COLLEGE

PH: 6674003/9264117 E-mail: basangam@connect.com.fj

Worksheet 12

School: Ba Sangam College **Year / Level:** 13

Subject: Mathematics **Name of student:** ______________________

Strand	4 – Trigonometry
Sub strand	4.4 – Application of Addition Formulae
Content Learning Outcome	Transform trigonometric expressions into simple ones

Transformation of trigonometric expressions of the form $ACos\ \theta + BSin\ \theta$

Ref. Yr 13 Mathematics Textbook Pg 96-101

Addition Formulae

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(i) $a\ Cos\ \theta \pm b\ Sin\theta = r\ Cos\ (\theta \pm \alpha)$

(ii) $a\ Cos\ \theta \pm b\ Sin\theta = r\ Sin\ (\theta \pm \alpha)$

where α is an angle to be found and r is the modulus i.e. $r = \sqrt{a^2 + b^2}$ and a and b are ***coefficients*** of $Cos\ \theta$ and $Sin\ \theta$ respectively.

EXAMPLE 1:

Express $y = 2\cos\theta + \sqrt{2}\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Answer

$a = 2, b = \sqrt{2}\ \therefore r = \sqrt{\sqrt{2}^2 + 2^2}$

$r = \sqrt{6}$

Apply Addition law: • $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

And substituting r yields: $\sqrt{6}\cos\ \theta Cos\ \alpha - \sqrt{6}\ Sin\ \theta Sin\ \alpha$

Thus in general form, $2\ Cos\ \theta + \sqrt{2}\ Sin\theta$

$= \sqrt{6}\ Cos\ \alpha\ cos\ \theta + \ - \sqrt{6}\ Sin\ \alpha\ Sin\ \theta$

Compare Sine & Cosine functions

$2 = \sqrt{6}\ Cos\ \alpha$

$\cos\alpha = \frac{2}{\sqrt{6}}$

$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

$\alpha = 35.26^o$

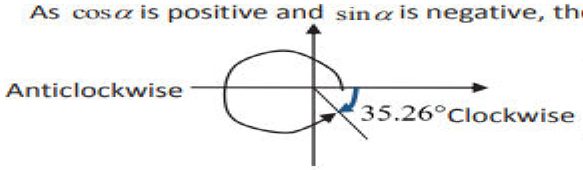
$\sqrt{2} = -\sqrt{6}\ Sin\ \alpha$

$\sin\alpha = \frac{\sqrt{2}}{-\sqrt{6}}$

$\alpha = \sin^{-1}\left(-\frac{\sqrt{2}}{\sqrt{6}}\right)$

$\alpha = -35.26^o$

As $\cos\alpha$ is positive and $\sin\alpha$ is negative, the angle lies in the 4th quadrant



Anticlockwise angle is positive
Clockwise angle is negative

$$\alpha = 360 - 35.26$$
$$= 324.74° \text{ or } -35.26°$$
$$\therefore \quad 2\cos\theta + \sqrt{2}\sin\theta = \sqrt{6}\cos(\theta - 35.26°)$$

EXAMPLE 2:

Using the previous example,

a) Give the coordinates of the maximum and minimum point on $y = 2\cos\theta + \sqrt{2}\sin\theta$ for $0 \le \theta \le 360°$

b) Find the x and $y-$intercepts.

c) Sketch the graph of $y = 2\cos\theta + \sqrt{2}\sin\theta$ for $0 \le \theta \le 360°$

✍ **Answers**

Since it was reduced to a single trig function i.e.

$2\cos\theta + \sqrt{2}\sin\theta = \sqrt{6}\cos(\theta - 35.26°)$, so it is easier to analyze

$$y = \sqrt{6}\cos(\theta - 35.26°)$$

a) Coordinates of the maximum and minimum point for $0 \le \theta \le 360°$

Cosine is maximum at $0°$

$$2\cos\theta + \sqrt{2}\sin\theta = \sqrt{6}\cos(\theta - 35.26°)$$
$$\theta - 35.26° = 0$$
$$\theta = 35.26°$$

The maximum occurs at $(35.26°, \sqrt{6})$

Cosine is minimum at $180°$

$$2\cos\theta + \sqrt{2}\sin\theta = \sqrt{6}\cos(\theta - 35.26°)$$
$$\theta - 35.26° = 180°$$
$$\theta = 215.26°$$

The minimum occurs at $(215.26°, -\sqrt{6})$

b) Intercepts:

Let's find the y-intercept,
By letting $\theta = 0$

$$y = \sqrt{6}\cos(\theta - 35.26°)$$
$$= \sqrt{6}\cos(0 - 35.26°)$$
$$= \sqrt{6}\cos(-35.26°)$$
$$\therefore y = 2$$

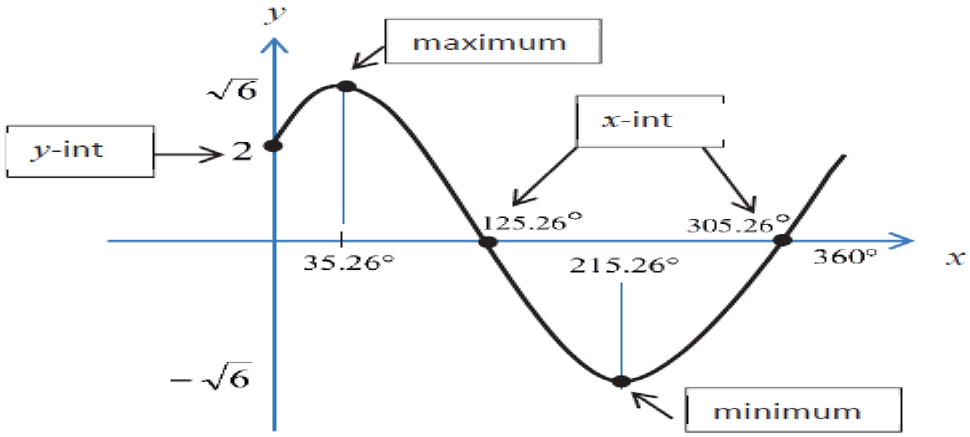
Let's find the x-intercept
By letting $y = 0$

$$y = \sqrt{6}\cos(\theta - 35.26°)$$
$$0 = \sqrt{6}\cos(\theta - 35.26°)$$

Cosine is 0 at $90°$ and $270°$

$$\theta - 35.26° = 90°$$
$$\theta = 125.26°$$
$$\theta - 35.26° = 270°$$
$$\theta = 305.26°$$
$$\therefore x = 125.26°, 305.26°$$

c) Graph of $y = 2\cos\theta + \sqrt{2}\sin\theta$ for $0 \le \theta \le 2\pi$ is shown below



ACTIVITY:

1.

A function is given by $f(x) = 7\sin x + 8\cos x$

a) Express the function $f(x)$ in the form $R\cos(x+\alpha)$ where α is an acute angle.

b) Hence, sketch the graph of $f(x) = 7\sin x + 8\cos x$ for $0 \le \theta \le 360°$

c) Solve the equation $7\sin x + 8\cos x = 6$ for $0 \le \theta \le 360°$ *(6 m)*

2.

A function is given by $f(x) = 7\cos x - 6\sin x$.

a) Express the function $f(x)$ in the form $f(x) = R\cos(x + \theta)$, where θ is an acute angle.

b) Hence, sketch the graph of $f(x) = 7\cos x - 6\sin x$ for $0 \le \theta \le 360°$

c) Solve the equation $7\cos x - 6\sin x = 5$ for $0 \le \theta \le 360°$

(6 m)