

Example 3:

Prove $\frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x} = \tan x$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\text{LHS} = \frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$= \frac{2\cancel{\cos 4x}\sin x}{2\cancel{\cos 4x}\cos x}$$

$$= \frac{\sin x}{\cos x} = \tan x = \text{RHS}$$

Exercise:

Prove $\frac{\sin 2\theta + \sin 5\theta}{\cos 2\theta - \cos 5\theta} = \cot \frac{3\theta}{2}$

Product to Sum Rule

<ul style="list-style-type: none"> • $2\sin A \cos B = \sin(A+B) + \sin(A-B)$ • $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ • $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

Example 1:

Write as sum of $2\sin 3x \cos x$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\sin 3x \cos x = \sin(3x+x) + \sin(3x-x)$$

$$= \sin 4x + \sin 2x$$

Example 2:

Write $\cos 30^\circ \cos 10^\circ$ as a sum

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

$$\cos 30^\circ \cos 10^\circ = \frac{1}{2}\cos(30+10) + \frac{1}{2}\cos(30-10)$$

Exercise:

1. Express each function as a sum

a) $\sin 5x \sin 3x$

b) $\cos 3x \sin 4x$

2. Prove the following $\frac{\cos \theta - \cos 3\theta}{\sin 3\theta - \sin \theta} = \tan 2\theta$

3. Express $\sin \frac{\pi}{4} \cos \frac{3\pi}{2}$ as a sum.