STRAND 4   **GRAPHS**
4.1   GRAPHS

Learning Objective
At the end of this lesson, students should be able to:
- Transform quadratic equation in the form $a(x \pm h)^2 \pm k$
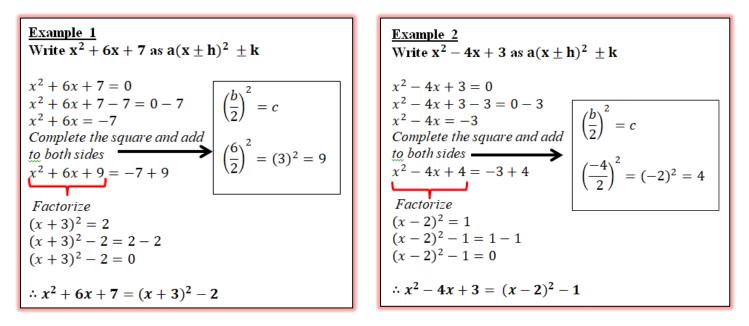

Quadratic Functions

- ❖ Quadratic equation has the degree (i.e. the highest power of the variable) equal to 2.
- ❖ All quadratic equations have a standard equation. That is:

$$y = ax^2 + bx + c$$

Where: a, b and c are constants
$a \neq 0$

Completing the Square

- ❖ The technique of completing the square is a method used to write quadratic expressions in perfect square form. That is a quadratic equation $ax^2 + bx + c = 0$ is written as $a(x \pm h)^2 \pm k = 0$

Example 1
Write $x^2 + 6x + 7$ as $a(x \pm h)^2 \pm k$

$x^2 + 6x + 7 = 0$
$x^2 + 6x + 7 - 7 = 0 - 7$
$x^2 + 6x = -7$
Complete the square and add to both sides →

$\left(\frac{b}{2}\right)^2 = c$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$x^2 + 6x + 9 = -7 + 9$

Factorize
$(x + 3)^2 = 2$
$(x + 3)^2 - 2 = 2 - 2$
$(x + 3)^2 - 2 = 0$

∴ $x^2 + 6x + 7 = (x + 3)^2 - 2$

Example 2
Write $x^2 - 4x + 3$ as $a(x \pm h)^2 \pm k$

$x^2 - 4x + 3 = 0$
$x^2 - 4x + 3 - 3 = 0 - 3$
$x^2 - 4x = -3$
Complete the square and add to both sides →

$\left(\frac{b}{2}\right)^2 = c$

$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$x^2 - 4x + 4 = -3 + 4$

Factorize
$(x - 2)^2 = 1$
$(x - 2)^2 - 1 = 1 - 1$
$(x - 2)^2 - 1 = 0$

∴ $x^2 - 4x + 3 = (x - 2)^2 - 1$

Example 3: Write $3x^2 - 2x + 5$ as $a(x \pm h)^2 \pm k$
Factorize to make the coefficient of x^2 equal to 1

$3\left(x^2 - \frac{2}{3}x + \frac{5}{3}\right)$

$3\left(x^2 - \frac{2}{3}x + \frac{5}{3}\right) = 0$

Complete the square and add to both sides
$\left(x^2 - \frac{2}{3}x + \frac{5}{3}\right) = 0$ →

$\left(\frac{b}{2}\right)^2 = c$

$\left(\frac{-2/3}{2}\right)^2 = \left(\frac{-1}{3}\right)^2 = \frac{1}{9} \rightarrow C$

$$x^2 - \frac{2}{3}x + \frac{5}{3} - \frac{5}{3} = 0 - \frac{5}{3}$$

$$x^2 - \frac{2}{3}x = -\frac{5}{3}$$

$$\underbrace{x^2 - \frac{2}{3}x + \frac{1}{9}}_{\text{Factorize}} = -\frac{5}{3} + \frac{1}{9}$$

Factorize

$$\left(x - \frac{1}{3}\right)^2 = -\frac{14}{9}$$

$$\left(x - \frac{1}{3}\right)^2 + \frac{14}{9} = -\frac{14}{9} + \frac{14}{9}$$

$$\left(x - \frac{1}{3}\right)^2 + \frac{14}{9} = 0$$

$$\therefore \ 3\left(x^2 - \frac{2}{3}x + \frac{5}{3}\right) = 3\left(x - \frac{1}{3}\right)^2 + \frac{14}{9}$$

Exercise: Write the following in the form $(x + a)^2 + b$

1. $\ x^2 - 8x + 5$ *2.* $\ x^2 + 10x + 30$ *3.* $\ 2x^2 + 4x - 3$