PENANG SANGAM HIGH SCHOOL

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LESSON NOTES – WEEK 14

School: Penang Sangam High School **Year/Level:** 13 **Subject:** Mathematics

Strand	4 **TRIGONOMETRY**
Sub Strand	4.1 Trigonometric Identities and Exact Values
Content Learning Outcome	Students should be able to; - prove identities using double angle formula - prove identities using half angle formula.

Double Angle Formula

• $\sin 2A = 2\sin A \cos A$	• $\cos 2A = 2\cos^2 A - 1$ $= \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A$	• $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

Example 1:

Prove that $\frac{1-cos\,2x}{1-sin^2 x} = 2\,tan^2 x$

LHS $= \frac{1-cos\,2x}{1-sin^2 x}$ $= 2 \times \frac{sin^2 x}{cos^2 x}$

$= \frac{1-(1-2sin^2 x)}{cos^2 x}$ $= 2 \times \left(\frac{\sin x}{\cos x}\right)^2$

$= \frac{1-1+2sin^2 x}{cos^2 x}$ $= 2\,tan^2 x$

$= \frac{2sin^2 x}{cos^2 x}$ =RHS

Example 2:

Prove that $\frac{2\,tan\,\theta}{1+tan^2\theta} = sin\,2\theta$

LHS $= \frac{2\,tan\,\theta}{1+tan^2\theta}$ $= tan^2\theta \times cos^2\theta$

$= \frac{2\,tan\,\theta}{sec^2\theta}$ $= 2\,\frac{\sin\theta}{\cos\theta} \times \cos\theta\,cos\theta$

$= \frac{2\,tan\,\theta}{\frac{1}{cos^2\theta}}$ $= 2\sin\theta\cos\theta$

$\cos 2A = cos^2\theta - sin^2\theta$ $= \sin 2\theta$

$cos^2\theta = \cos 2A + sin^2\theta$ = RHS

$= 2\tan\theta \div \frac{1}{\cos 2A + sin^2\theta}$

$= 2\tan\theta \div \frac{1}{\cos 2A + sin^2\theta}$

$= 2\tan\theta \times (1 - 2sin^2\theta) + sin^2\theta$

$= 2\tan\theta \times (1 - sin^2\theta)$

Exercise:

1. **Prove**
   a) $cosec\,2\theta - \cot 2\theta = tan\theta$

   b) $\frac{1-cos\,2\theta}{sin\,2\theta} = tan\theta$

• $\sin\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1-\cos A}{2}}$	• $\cos\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1+\cos A}{2}}$	• $\tan\left(\frac{A}{2}\right)=\frac{1-\cos A}{\sin A}$

- The half – angle formula can be derived from double angle formula.

Consider $\cos 2A = 2\cos^2 A - 1$

Let $A = \frac{\theta}{2}$

$\cos 2\left(\frac{\theta}{2}\right) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$

$\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$

$\cos\theta + 1 = 2\cos^2\left(\frac{\theta}{2}\right)$

$\cos^2\left(\frac{\theta}{2}\right) = \frac{\cos\theta + 1}{2}$

Consider $\cos 2A = 1 - 2\sin^2 A$

Let $A = \frac{\theta}{2}$

$\cos 2\left(\frac{\theta}{2}\right) = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$

$\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$

$2\sin^2\left(\frac{\theta}{2}\right) = 1 - \cos\theta$

$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$

Example 1:

Show that $\tan 22.5° = \sqrt{2} - 1$

$22.5° = \frac{45°}{2}$

$\tan 22.5° = \tan\frac{45°}{2}$

$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1-\cos 45°}{\sin 45}$

$= \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = (1 - \frac{1}{\sqrt{2}}) \div (\frac{1}{\sqrt{2}})$

$= (1 - \frac{1}{\sqrt{2}}) \times (\frac{\sqrt{2}}{1})$

$= (\frac{1\times\sqrt{2}}{1\times\sqrt{2}} - \frac{1}{\sqrt{2}}) \times (\sqrt{2})$

$= \frac{\sqrt{4}-\sqrt{2}}{\sqrt{2}} = \frac{2}{\sqrt{2}} - 1$

$= \left(\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\right) - 1 = \frac{2\sqrt{2}}{2} - 1 = \sqrt{2} - 1$

Example 2:

Find exact value of $sin\ 15°$

$sin\ 15° = \sin\frac{30°}{2}$

$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}} \quad \leftrightarrow \quad \sin\left(\frac{30}{2}\right) = \sqrt{\frac{1-\cos 30}{2}}$

$sin\ 15° = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2} \div 2}$

$= \sqrt{\frac{2-\sqrt{3}}{2} \div \frac{1}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{\sqrt{4}}}$

$sin\ 15° = \sqrt{\frac{2-\sqrt{3}}{2}}$

Exercise:

Find the values of the following in the simplest surd form.

1. $\cos 15°$
2. $\sin 22.5°$