Worksheet 16

School: Ba Sangam College   **Year / Level: 13**
Subject: Mathematics   **Name of student: _____**

Strand	6 – Algebra
Sub strand	6.4 – Partial Fractions
Content Learning Outcome	Decompose rational functions into a sum of simple functions

Partial Fractions   -   Quadratics Which Cannot be factorized
Ref. Yr 13 Mathematics Textbook *pg. 144 – 152*

Quadratic irreducible factor	$(x^2 + 4)$	$\dfrac{Ax + B}{(x^2 + 4)}$

Example 1

Express $\dfrac{x^2 - 1}{(x^2 + 1)(x - 2)}$ as a sum of partial fractions.

$$\frac{x^2 - 1}{(x^2 + 1)(x - 2)} = \frac{Ax + B}{(x^2 + 1)} + \frac{C}{(x - 2)}$$

$$\frac{x^2 - 1}{(x - 2)(x^2 + 1)} = \frac{(x - 2)(Ax + B) + C(x^2 + 1)}{(x - 2)(x^2 + 1)}$$

Since the denominator on both sides is same, we can equate the numerators.

$$x^2 - 1 = (x - 2)(Ax + B) + C(x^2 + 1)$$

Let $x = 2$,

$2^2 - 1 = (2-2)(A \times 2 + B) + C(2^2 + 1)$

$3 = 5C$

$C = \dfrac{3}{5}$

Let $x = 0$,

$0^2 - 1 = (0-2)(A(0) + B) + C(0^2 + 1)$

$-1 = -2B + C$

$-1 = -2B + \dfrac{3}{5}$

$B = \dfrac{4}{5}$

Let $x = 1$,

$(1)^2 - 1 = (1-2)(A + B) + C(1^2 + 1)$

$0 = -1(A + \dfrac{4}{5}) + 2\left(\dfrac{3}{5}\right)$

$= -A - \dfrac{4}{5} + \dfrac{6}{5}$

$0 = -A + \dfrac{2}{5}$

$A = \dfrac{2}{5}$

Hence $\dfrac{x^2 - 1}{(x^2 + 1)(x - 2)} = \dfrac{2x + 4}{5(x^2 + 1)} + \dfrac{3}{5(x - 2)}$

Example 2

Express $\dfrac{2x + 1}{(x - 2)(x^2 + 1)}$ as a sum of partial fractions.

Make denominators same

$$\frac{2x+1}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x-2)}{(x-2)(x^2+1)}$$

Since the denominator on both sides is same, we can equate the numerators

$$2x+1 = A(x^2+1)+(Bx+C)(x-2)$$

Let $x=2$,

$$2(2)+1 = A((2)^2+1)+(B(2)+C)(2-2)$$

$$5=5A \quad \Rightarrow A=1$$

Let $x=0$,

$$2(0)+1 = 1((0)^2+1)+(B(0)+C)(0-2)$$

$$1=1-2C \quad \Rightarrow C=0$$

Let $x=1$,

$$2(1)+1 = A((1)^2+1)+(B(1)+C)(1-2)$$

$$\Rightarrow 3=2A-B-C \quad \Rightarrow 3=2(1)-B+0 \quad \text{(recall that A =1 and C = 0)}$$

$$3=2-B \quad \Rightarrow B=-1$$

Therefore

$$\frac{2x+1}{(x-2)(x^2+1)} = \frac{1}{x-2} - \frac{x}{(x^2+1)}$$

ACTIVITY (3 marks each)

1. Express as a sum of partial fractions $$\frac{-x-2}{x(x^2+1)}$$
2. Express as a sum of partial fractions $$\frac{10x+24}{(x-3)(x^2+9)}$$