PENANG SANGAM HIGH SCHOOL
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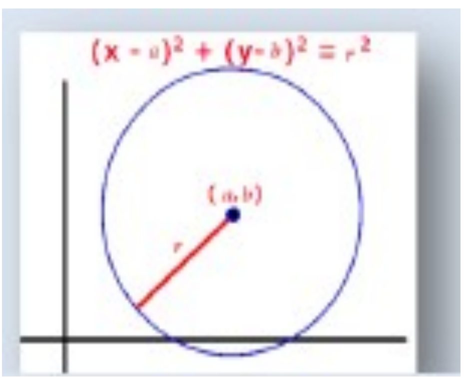
LESSON NOTES WEEK 16-18

Year/Level: 12 **Subject:** Mathematics

Strand	3 GRAPHS
Sub Strand	3.1.1 study and interpret graphs
Content Learning Outcome	Students should be able to: • Sketch the circles centered at the origin or at any givenpoint on a Cartesian plane. • Write equations of given circles. • State the domain and range

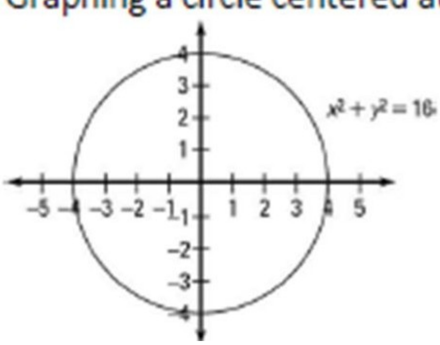
Lesson notes Topic: Graphs of circles

Equation of a circle:

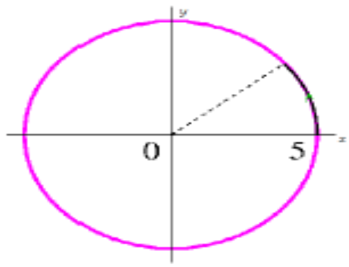
- $x^2 + y^2 = r^2$ is a circle with the centre at the origin(0, 0) and radius of size r.
- $(x - a)^2 + y - b^2 = r^2$ is a circle with the centre at the point (a, b) and radius of size r.
- 

EXAMPLE 1: Sketch the graph of

Graphing a circle centered at the o



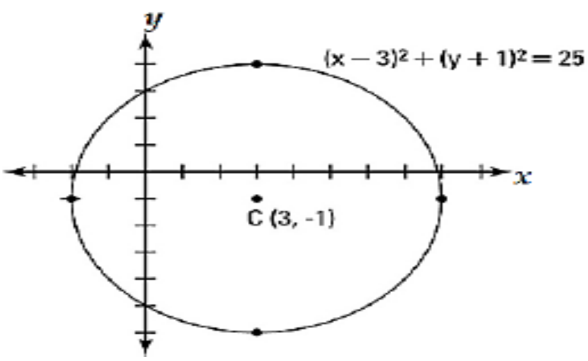
EXAMPLE 2: Write the equation of the graph



r = 5

$$x^2 + y^2 = r^2 = 5^2$$
$$\therefore x^2 + y^2 = 25$$

EXAMPLE 3: Sketch the graph of $(x-3)^2 + (y+1)^2 = 25$



Activity

1. What is the range of the relation $x^2 + y^2 = 4$

   A. $y \le 4$
   B. $-4 \le y \le 4$
   C. $-2 \le y \le 2$
   D. y = real numbers

2. The coordinates of the centre of the circle $(x - 3)^2 + y^2 = 4$ are:

   A. (3, 0) B. (-3, 0) C. (3, 4) D. (-3, 4)

3. A circle is defined by the equation $(x - 4)^2 + (y - 2)^2 = 9$. The value of the radius of this circle is

   A. 2 units B. 4 units C.3 units D. 9 units

4. For the equation $y^2 + x^2 = 9$

   a. Sketch the graph
   b. State the domain
   c. State the range.

5. The coordinates of the end points of the diameter of a circle are (-5, 0) and (5, 0)

   a. Write down the equation of this circle.
   b. Determine the domain and range.

Strand	3 graphs
Sub Strand	3.1.2 interpret and solve simultaneous equations
Content Learning Outcome	Students should be able to: • Solve equations simultaneously. • Apply the concept on word problems.

Lesson notes Topic: Application of simultaneous equations

Recap: Three methods learnt in year 11 are:

- **Elimination method**: one variable is eliminated either by adding or subtracting the two equations.
- **Substitution method**: one equation is substituted into another equation and solved for the two variables.
- **Graphical method**: both the linear graphs are accurately drawn and the point of intersection is the solution.(not encouraged due to a lot of approximation)

Example 1 solve this pair of equation:

$\mathbf{3x - 2y = 8}$ (1)

$\mathbf{2x - y = 5}$ (2)

Solution: (using elimination method)

$\mathbf{3x - 2y = 8}$ $(\times\ -1)$ $\mathbf{-3x + 2y = -8}$

$\mathbf{2x - y = 5}$ $(\times\ 2)$ $\mathbf{4x - 2y = 10}$

Adding: $x = 2$

Replacing (x=2) in equation (1)

$3(2) - 2y = 8$

$6 - 2y = 8$

$-2y = 2$

$y = -1$

Hence: the solution is $\boldsymbol{x = 2\ and\ y = -1}$

Example 2:Application on word problems

Steps to answer word problems

Step 1. **Read** the problem. Make sure you understand all the words and ideas. You may need to read the problem two or more times.
Step 2. **Identify** what you are looking for. It's hard to find something if you are not sure what it is! Read the problem again and look for words that tell you what you are looking for!
Step 3. **Name** what you are looking for. Choose a variable to represent that quantity. You can use any letter for the variable, but it may help to choose one that helps you remember what it represents.

Step 4. **Translate** into an equation. It may help to first restate the problem in one sentence, with all the important information. Then translate the sentence into an equation.
Step 5. **Solve** the equation using good algebra techniques. Even if you know the answer right away, using algebra will better prepare you to solve problems that do not have obvious answers.
Step 6. **Check** the answer in the problem and make sure it makes sense. **Answer** the question with a complete sentence.

Problem

The sum of two number is 14 and their difference is 2. Find the numbers.

Solution:

Let the two numbers be x and y.

x + y = 14............... (i)

x - y = 2................(ii)

Adding equation (i) and (ii), we get 2x = 16

$$\frac{2x}{2} = \frac{16}{2}$$

Numbers 1, 2 and 3

Hence, the two numbers are 6 and 8.

Activity

Exercise 36

Page 119

x = 8

Substituting the value x in

equation (i), we get8 + y = 14

or, 8 – 8 + y = 14 - 8

o

r,

y

=

1

4 - 8or, y = 6

Therefore, x = 8 and y = 6

Strand	3 graphs
Sub Strand	3.1.2 interpret and solve simultaneous equations
Content Learning Outcome	Students should be able to: • Find the points of intersection for a linear function and quadratic function. • Find the points of intersection for a linear function and hyperbolic function.

Lesson notes <u>Topic: Points of intersection</u>

- Use substitution method.
- Simplify or collect all terms on one side.
- Factorize and solve.(you might use null factor law)

1. **Linear and quadratic equations**

Example: Find the coordinates of the points of intersection of the line $y = 2x + 1$ and the parabola $y = x^2 - 2$

$y = 2x + 1$ and $y = x^2 - 2$

$x^2 - 2 = 2x + 1$

$x^2 - 2 - 2x - 1 = 0$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

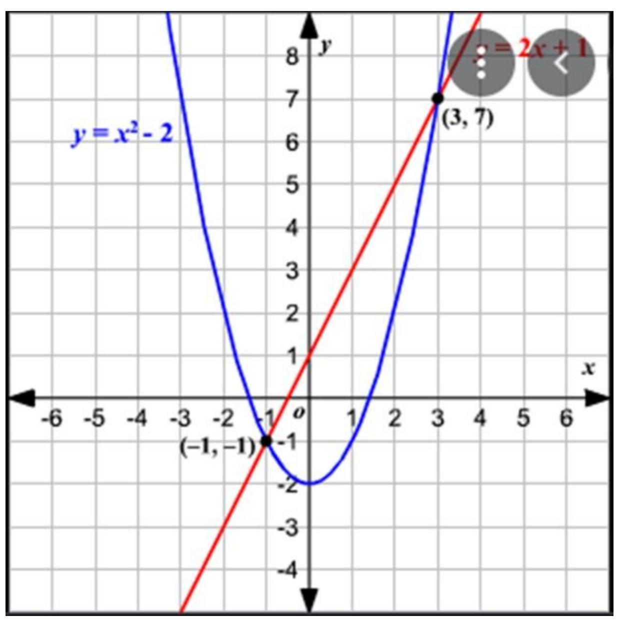
$x \in \{-1, 3\}$

Null factor law

Type I factorization

$-3 \times 1 = -3$

$-3 + 1 = -2$



Substitute the x value in any equation:

$if\, x = -1 = 2x + 1 = 2(-1) + 1 = -1$

$if\, x = 3 = 2x + 1 = 2(3) + 1 = 7$

Therefore: $(\mathbf{-1, -1})$ $(\mathbf{3, 7})$

2. **Linear and hyperbolic equation:** will meet at two places.

Example: Find the coordinates of the points of intersection of the line $y = 2x - 1$ and the hyperbola $y = \frac{5}{x-2}$.

$y = 2x - 1$ and $y = \frac{5}{x-2}$

$2x - 1 = \frac{5}{x - 2}$

$(2x - 1)(x - 2) = 5$

Expansion

$2x^2 - 4x - x + 2 = 5$

$2x^2 - 5x - 3 = 0$

Type ii Factorization

$2 \times -3 = -6$

$-6 \times 1 = -6$

$-6 + 1 = -5$

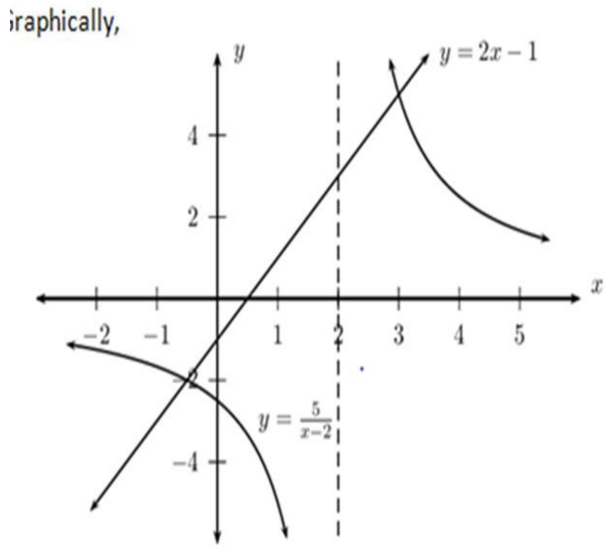
$2x^2 - 6x + x - 3 = 0$ $(2x$

$^2 - 6x)(+x - 3) = 0$

$2(x - 3) + 1(x - 3) = 0$

$(2x + 1)(x - 3) = 0$

$x \in \{\frac{-1}{2}, 3\}$



Substitute the x value in any equation:

$if\ x = \frac{-1}{2} = 2x - 1 = 2\left(\frac{-1}{2}\right) - 1 = -1$

$if\ x = 3 = 2x - 1 = 2(3) - 1 = 5$

Therefore: $(\frac{-}{2}, -1)$ ***and*** $(\mathbf{3}, \mathbf{5})$

Activity

1. Find the coordinates of the points of intersection of the line $y = 3x + 1$ and the parabola $y = x^2 - 3$.

2. Find the coordinates of the points of intersection of the line $y = x - 3$ and the curve $y = -\frac{2}{x}$.

Strand	3 graphs
Sub Strand	3.1.2 interpret and solve simultaneous equations
Content Learning Outcome	Students should be able to: • Find the points of intersection for a linear function and circles.

Lesson notes <u>Topic: Points of intersection</u>

- Use substitution method.
- Simplify or collect all terms on one side.
- Factorize and solve.(you might use null factor law)

3. **Linear equations and circles:** meets at two places.

Example: Find the points of intersection for the equations $y^2 + x^2 = 5$ and $= 3x - 5$.

$y^2 + x^2 = 5$ and $y = 3x - 5$

$(3x - 5)^2 + x^2 = 5 \longrightarrow$ Expansion

$9x^2 - 30x + 25 + x^2 = 5 \longrightarrow$ Simplify like terms

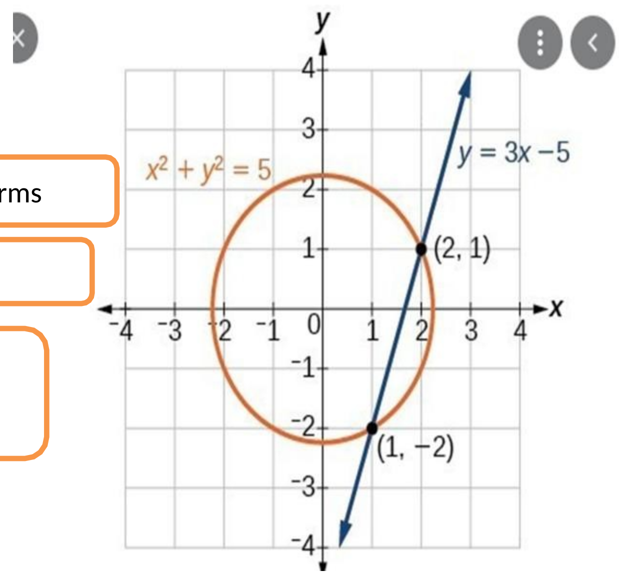
Factorization

$10x^2 - 30x + 20 = 0$

$10(x^2 - 3x + 2) = 0 \longrightarrow$ Factorization Type I

$\longrightarrow$ any equation

$(x - 2)(x - 1) = 0$



$x \in$ *2, 1+

Substitute the x value in

If $x = 2, y = 3x - 5, 3(2) - 5 = 1$

$x = 1, y = 3x - 5, 3(1) - 5 = -2$

Therefore: $(\mathbf{1}, -\mathbf{2})$ $(\mathbf{2}, \mathbf{1})$

Activity

1. Find the points of intersection for the equations $x + y = 3$ and $y^2 + x^2 = 9$.

<u>2015</u>

24. Find the points of intersection of the functions $y = 2x + 1$ and $y = x^2 + 1$. **(2 marks)**

2016

29. Find the **coordinates** of the point of intersection of the functions $y = x + 2$ and $y = \frac{-1}{x}$ **(2 marks)**

Strand	4 COORDINATE GEOMETRY
Sub Strand	4.1.1 explore and apply concepts on coordinate geometry
Content Learning Outcome	Students should be able to: • Calculate the distance, midpoint and gradient. • Apply these concepts.

Strand 4: Coordinate Geometry

Lesson notes

Review of formulae

- Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoints: $M_{(x_m, y_m)} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- Gradients/slopes: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
- Equation of a line: $y = mx + c$ or $y - y_1 = m(x - x_1)$
- Gradient with angle: $m = \tan\theta$

Example 1 What is the gradient of the line $\frac{x}{3} - \frac{y}{5} = 2$

Solution: make y the subject of the formula and write the equation in the general form $\boldsymbol{y = mx + c}$

Example 2 Find the equation of a line through the point (0,-4) which makes an angle of 135°with the positive x-axis.

Solution Equation

$m = \tan\theta$ $y - y_1 = (x - x_1)$

$\frac{x}{3} - \frac{y}{5} = 2$

$5x - 3y$

$\overline{\quad 15 \quad} = 2$

$5x - 3y = 30$

$\frac{-3y}{-3} = \frac{-5x + 30}{-3}$

$-5x \quad 30$

$y = \overline{-3} + \overline{-3}$

$5x - \underline{5}x - 3y$

$= 30 -5x$

$m = \tan 135°$

$\boldsymbol{m = -1}$

$y - -4 = -1(x - 0)$

$y + 4 = -1(x)$

$y + 4 - 4 = -1(x) - 4$

$5x$

$y = \quad - 103$

Therefore: $m = 5$ 3 $\boldsymbol{y = -x - 4}$ –

Example 3 A diameter intersects the circumference of a circle at the points (-3, 3) and (1, 0). Find

a. length of the diameter.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(1 - -3)^2 + (0 - 3)^2}$$

$$d = \sqrt{(4)^2 + (-3)^2}$$

$$d = \sqrt{25}$$

$$\boldsymbol{d = 5\ units}$$

b. the length of the radius.

$$radius = \frac{diameter}{2}$$

$$radius = \frac{5}{2}$$

$$\boldsymbol{radius = 2.5\ units}$$

c. find the coordinates of the center of the circle.

$$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

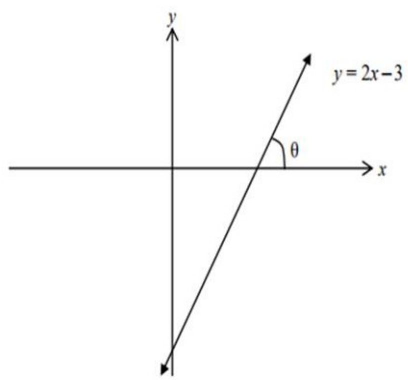
$$midpoint = \left(\frac{-3 + 1}{2}, \frac{3 + 0}{2}\right)$$

$$\boldsymbol{midpoint = \left(-1, \frac{3}{2}\right)}$$

Activity

2018

2. The value of angle, θ, the line $y = 2x - 3$ makes with the **positive** x-axis is



A. $\tan^{-1}(3)$

B. $\tan^{-1}(2)$

C. $\tan^{-1}\left(\frac{3}{2}\right)$

D. $\tan^{-1}\left(\frac{2}{3}\right)$

(1 mark)

SANGA

2. A line has gradient -3. Calculate the angle between the line and the positive x-axis.

3. For the given pair of points

(i). (-3,2) and (1, 6)

(ii). (-2,-5) and (5, 9)

Find :

a. distance
b. midpoint
c. gradient
d. equation.

e. angle that makes with the positive x

Strand	4 COORDINATE GEOMETRY
Sub Strand	4.1.1 explore and apply concepts on coordinate geometry
Content Learning Outcome	Students should be able to: • Calculate the gradients of parallel and perpendicular lines. • Apply these concepts and write equations of lines.

Strand 4: Coordinate Geometry

Lesson notes **Parallel and Perpendicular lines**

Parallel lines: are two lines on the same plane and they do not intersect.

- they have same slope/gradients.
- If m_1 is the gradient of the first line and m_2 is the gradient of the second line then it is parallel if $m_1 = m_2$.

Example

Find the equation of the line through the point (0, -2) and is parallel to $3y = 6 - 2x$

Make y the subject to find m.

$3y = 6 - 2x$

$\frac{3y}{3} = \frac{6 - 2x}{3}$

$y = \frac{6}{3} - \frac{2x}{3}$

$y = 2 - \frac{2x}{3}$

Therefore $m = \frac{-2}{3}$

Given a point (0, -2) and $m = \frac{-2}{3}$

Using the equation of a line formula:

$y - y_1 = (x - x_1)$

$y - -2 = \frac{-2}{3}(x - 0)$

$y + 2 = \frac{-2}{3}(x)$

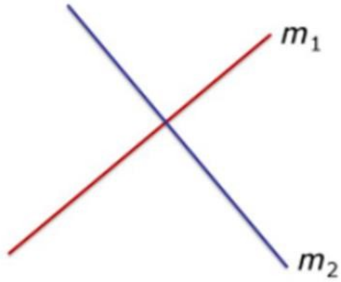
$y + 2 - 2 = \frac{-2}{3}(x) - 2$

$\boldsymbol{y = \frac{-2x}{3} - 2}$

Perpendicular lines- means at right angles or meeting at 90°.

- Two lines are perpendicular if the product of their gradients is equal to-1., ie, $m_1 \times m_2 = -1$

If two lines are perpendicular to each other with gradients m_1 and m_2 then it is true to say



$$m_1 = -\frac{1}{m_2}$$

or

$$m_1 \times m_2 = -1$$

Example: Find the equation of the line through the point (2,1) and is perpendicular to the line joining points (-3,4) and (2,1).

Gradients between (-3,4) and (2,1).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{4 - 1}{-3 - 2}$$

$$m_1 = \frac{\mathbf{-3}}{\mathbf{5}}$$

Since this line is perpendicular to the second line, then the gradient of second line is :

$$m_2 = \frac{-1}{m_1}$$

$$m_2 = \frac{5}{3}$$

Given a point (2, 1) and $m = \frac{5}{3}$

Using the equation of a line formula:

$$y - y_1 = (x - x_1)$$

$$y - 1 = \frac{5}{3}(x - 2)$$

$$y - 1 + 1 = \frac{5x}{3} - \frac{10}{3} + 1$$

$$\boldsymbol{y = \frac{5x}{3} - \frac{7}{3}}$$

<u>Homework</u>

Exercise 39

1, 5, and 6

Strand	4 COORDINATE GEOMETRY
Sub Strand	4.1.1 explore and apply concepts on coordinate geometry
Content Learning Outcome	Students should be able to: • Calculate the gradient and identify collinear points.

Strand 4: Coordinate Geometry

Lesson notes **Collinear points**

- These points lie on the same line.
- The gradients between each pair of lines will be the same.
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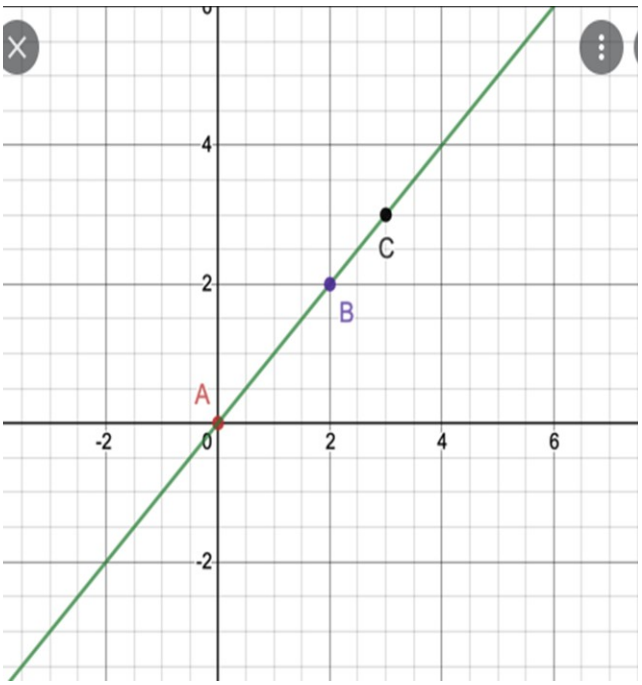


Consider points A, B and C.

They are collinear and so their gradients will be same.

Example

Given points $(1,2)$, $B(2,4)$ and $C(3,x)$, then find the value of x.

Find the gradient of AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-2}{2-1}$$

$$m = 2$$

Find the gradient of BC

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$m = \frac{x-4}{3-2}$ (substitute $m = 2$)

$$2 = \frac{x-4}{1}$$

$$2 = x - 4$$

$$2 + 4 = x - 4 + 4$$

$$\mathbf{6 = x}$$

Activity

2017

4. The points (3, 4), (8, 19) and (9, **p**) are **collinear**, that is they lie on the **same line**.

   Calculate the value of **p**. **(2 marks)**

2018

4. Show that the following points are **collinear**:

   (1, 4), (4, 6) and (10, 10)

   (2 marks)

2019

1. Which of the following pairs of lines is **parallel**?

   A. $y - 2x = 0$ and $y - \frac{1}{2}x = 0$

   B. $y - 2x = 0$ and $y - 2x + 5 = 0$

   C. $y - 3x + 5 = 0$ and $y + 3x + 5 = 0$

   D. $y - 2x + 5 = 0$ and $y + \frac{1}{2}x + 5 = 0$ **(1 mark)**

Additional question

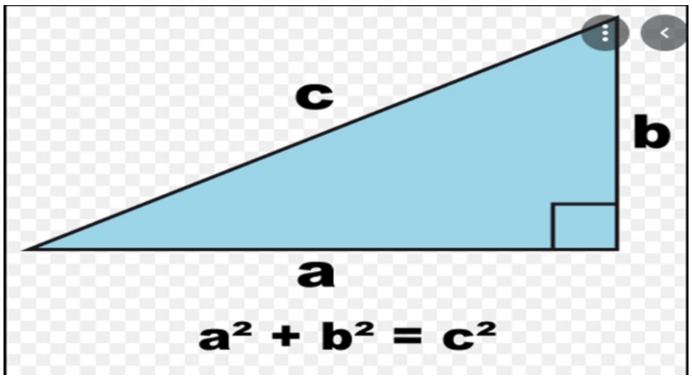
Find out whether the points P(1, 2), Q(2, 3), and R(3, 4) are collinear or not.

Strand	5 TRIGNOMETRY
Sub Strand	5.1.1 investigate and solve problems using trignometry
Content Learning Outcome	Students should be able to: • Solve right and non - right angled triangles. • Apply Pythagoras theorem on right angled triangle. • Use SOH/CAH/TOA rules on right angled triangles.

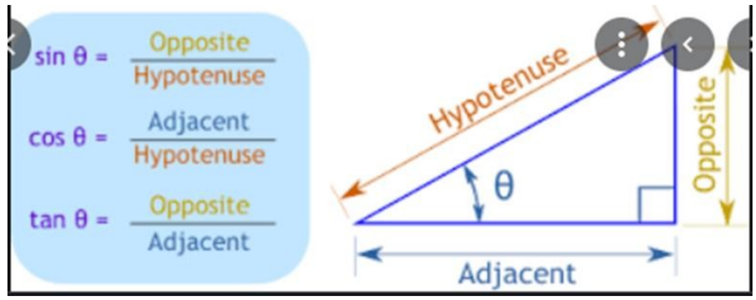
LESSON NOTES **STRAND 5: TRIGONOMETRY**

Right Angled Triangle

Pythagoras theorem:

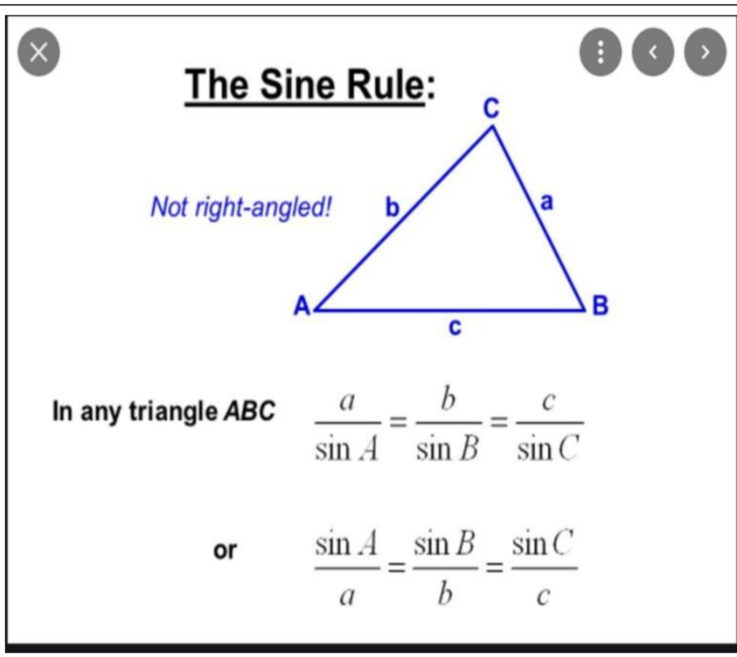


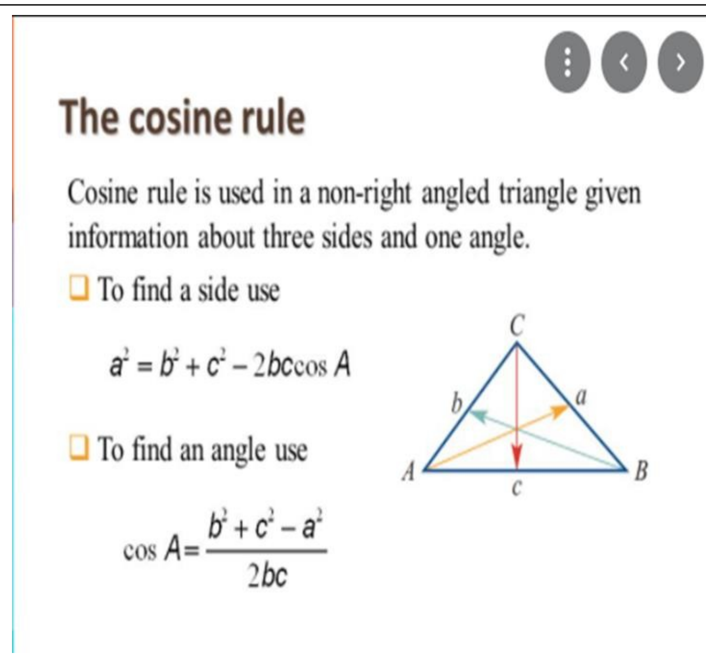
SOH/CAH/TOA



NON – RIGHT ANGLED TRIANGLE

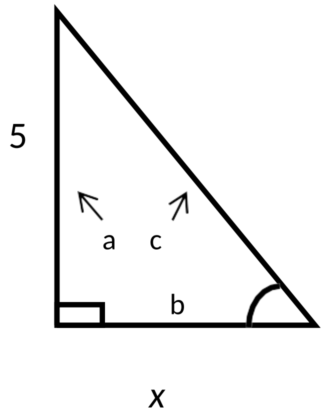
The **angles** are labeled with **capital letters**. The **opposite sides** are labeled with **lower case** letters. Notice that an angle and its opposite side are the same letter.





Example 1 (Pythagoras Theorem)

In the right triangle given below, find the value of **cos *θ***.



First, find hypotenuse:

$$c^2 = a^2 + b^2$$

$$c^2 = 5^2 + x^2$$

$$c = \sqrt{25 + x^2}$$

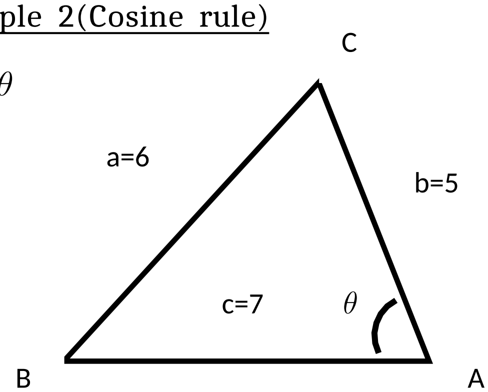
$$hypotenuse = \sqrt{25 + x^2}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse}$$

$$\boldsymbol{\cos\theta = \frac{x}{\sqrt{25 + x^2}}}$$

Example 2(Cosine rule)

Find θ



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{5^2 + 7^2 - 6}{2(5)(7)}$$

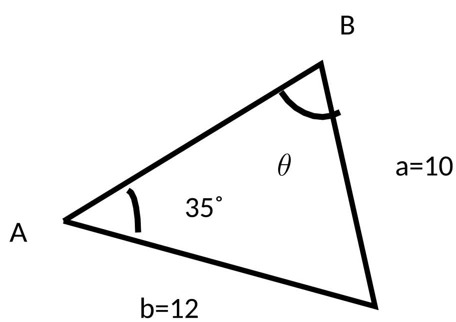
$$\cos\theta = \frac{38}{70}$$

$$\theta = \cos^{-1}\frac{38}{70}$$

$$\boldsymbol{\theta = 57.12°}$$

Example 3(Sine Rule)

Find θ



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 35°}{10} = \frac{\sin\theta}{12}$$

$$\frac{12\sin 35°}{10} = sin\ \theta$$

$$\theta = \sin^{-1}\left(\frac{12\sin 35°}{10}\right)$$

$$\theta = 43.50°$$

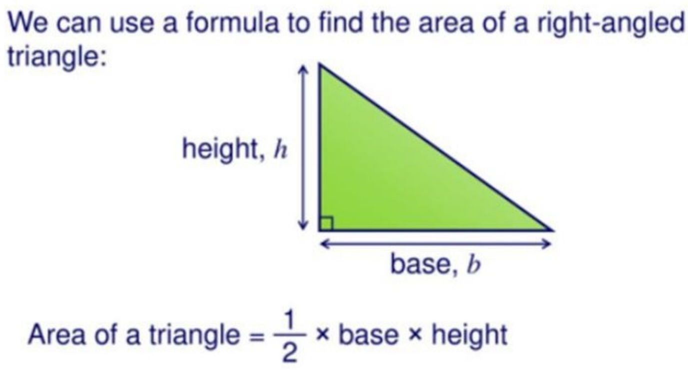
ACTIVITY

Exercise 40 Page 144

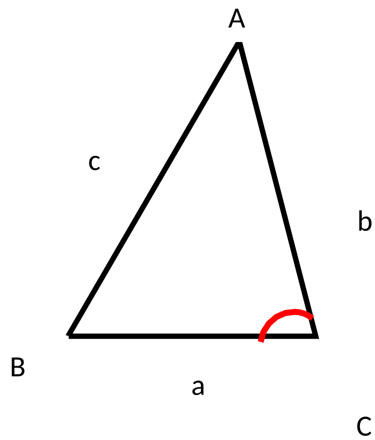
Numbers 2 and 3

Strand	5 TRIGNOMETRY
Sub Strand	5.1.1 investigate and solve problems using trignometry
Content Learning Outcome	Students should be able to: • Find the area of right angled and non- right angled triangles..

<u>Right angled triangle</u>



<u>Non right angled triangle</u>: the area can be found if two sides and an angle is given.



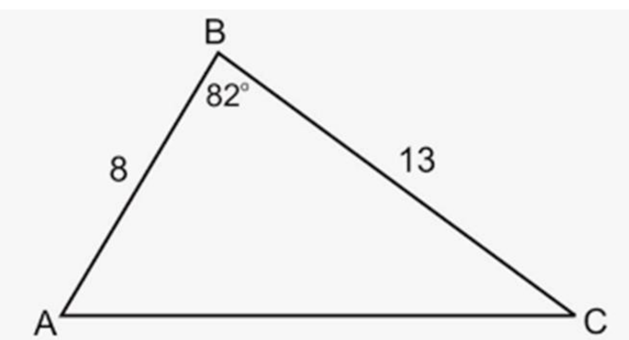
$$Area = \frac{1}{2}ab\sin C$$

$$A = \frac{1}{2} \times \text{two known side} \times \sin(\text{known angle})$$

<u>Examples:</u> Find the area of each non right angled triangle

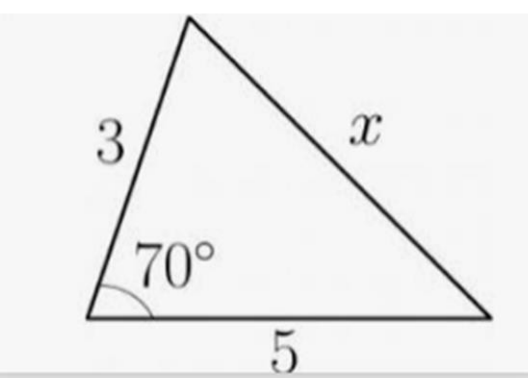
a.

$$area = \frac{1}{2}ac\sin B$$ (please note we know angle B)



$$= \frac{1}{2}(8)(13)(\sin 82°)$$

$$= 51.49 \text{ sq. units}$$



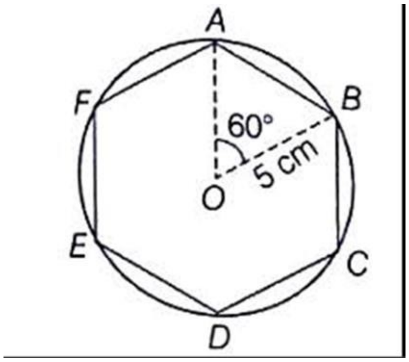
$A =$

1

$$\frac{}{2}(3)(5)\sin 70$$

$$= \mathbf{7.05}\ \boldsymbol{sq.units}$$

Example 3 The diagram below shows a regular hexagon inscribed in a circle of radius 5cm at center O.



a. $Area = \frac{1}{2} absin\ C$

$= \frac{1}{2}(5 \times\ 5)\sin 60°$

$= \mathbf{10.83} \boldsymbol{sq.\ units}$

b. area of hexagon

$10.83\ \times\ 6 = \mathbf{64.95}\ \boldsymbol{sq.\ units}$

a) The area of one of the triangle.
b) The area of the hexagon.

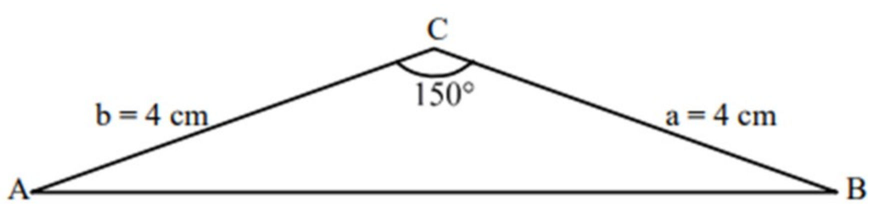
Activity

Exercise 41 page 149

Numbers :1 and 3

2017 paper

2. An **isosceles** triangle is shown below.



The **area** of this triangle in cm^2 is

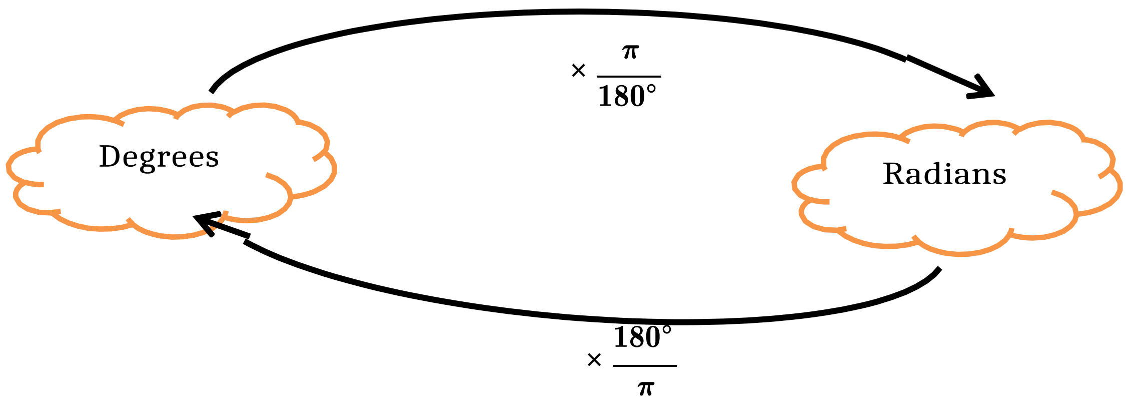
A. 4

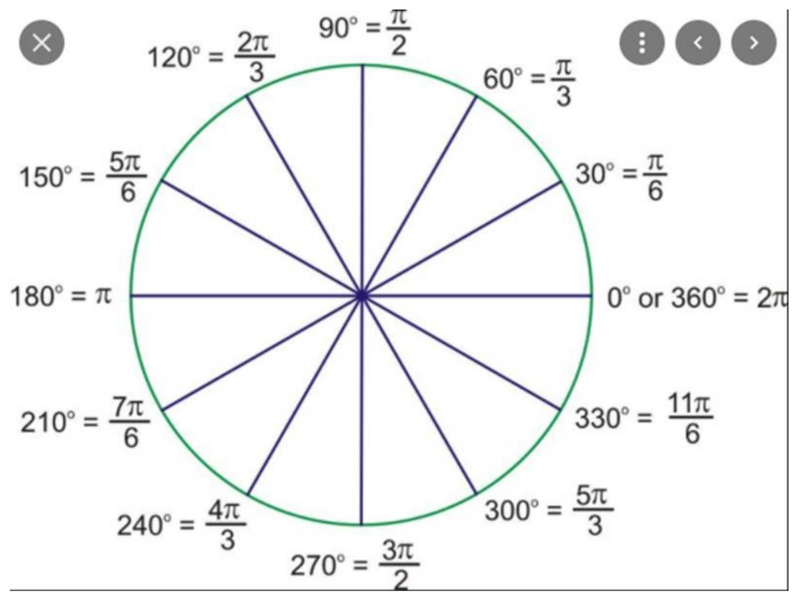
B. 6

C. 8

Strand	5 TRIGNOMETRY
Sub Strand	5.1.1 investigate and solve problems using trignometry
Content Learning Outcome	Students should be able to: • Convert angles from degrees to radians and vice versa. • Find arc length, area of sector and segment

Lesson notes **Conversion of degrees to radians and vice versa**





Example: convert 45° to radians.

$$45° \times \frac{\pi}{180°} = \frac{\pi}{4} \text{ or } 0.79 \text{ rads}$$

Example: convert $\frac{\pi}{5}$ rads to degrees.

$$\frac{\pi}{5} \times \frac{180°}{\pi} = 36°$$

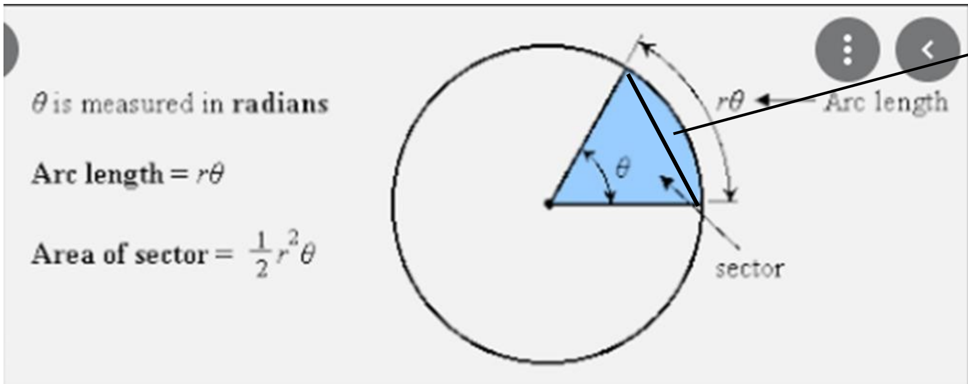
Exercise:

1. Convert 15° to radians.

2. Convert $\frac{5\pi}{2}$ radians to degrees.

Strand	5
Sub Strand	5.1.1
Content Learning Outcome	Students should be able to: • Convert angles from degrees to radians and vice versa. • Find arc length, area of sector and segment

Lesson notes **Arc length, Area of sector and segment**



Area of segment = Area of sector – area of triangle

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$

radians Degrees

Example

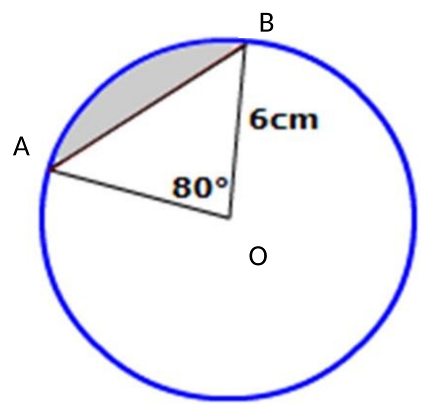
The diagram below shows a circle of radius 6cm.

a. Convert the angle to radians.

b. Find the length of the arc AB.

c. Find the area of sector OAB.

d. Find the area of segment



Solutions

a. $80° \times \frac{\pi}{180°}$

$= 1.396\ rads$

b. Arc length$= r\theta$

$= 6 \times 1.396$

$= 8.376$

$= \mathbf{8.38\ cm}$

c. $\boldsymbol{A = \frac{1}{2}r^2\theta}$

$= \frac{1}{2}(6^2)1.396$

$= \mathbf{25.13cm^2}$

d.

$\boldsymbol{A = \frac{1}{2}r^2(\theta - \sin\theta)}$

$= \frac{1}{2}(6^2)(1.396 - \sin 80°)$

$= \mathbf{7.40cm^2}$

<u>Activity</u>

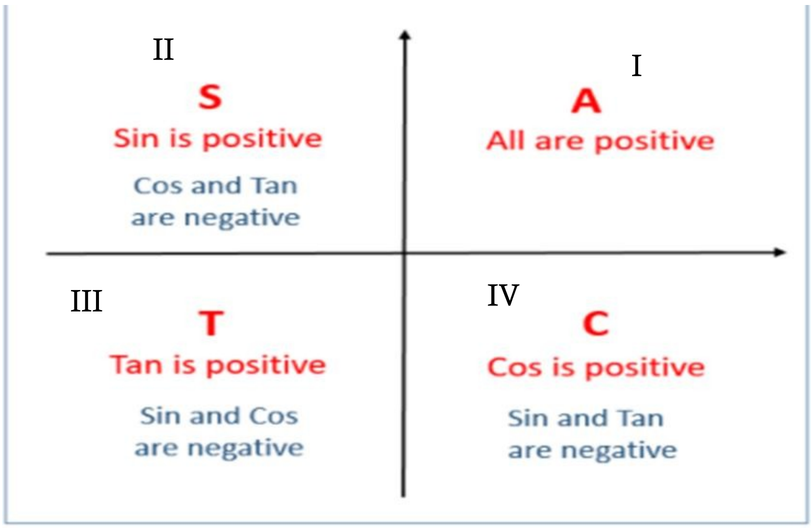
Exercise 42 page 153

No. 1 and 2

Strand	5 TRIGNOMETRY
Sub Strand	5.1.1 investigate and solve problems using trignometry
Content Learning Outcome	Students should be able to: • Solve trigonometric equations in the required domain.

Lesson notes **Trigonometric equations**

Recap: Quadrant diagram



Example 1: Solve $\tan\theta - 1 = 0, 0° \leq \theta \leq 360°$

$\tan\theta - 1 = 0$

$\tan\theta - 1 + 1 = 0 + 1$

Tan is positive in quadrant I and III

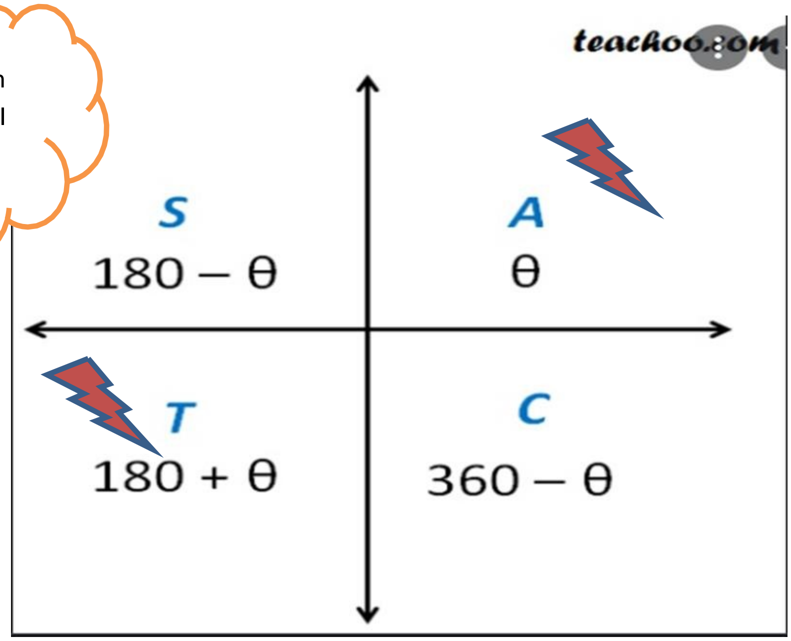
$\tan\theta = +1$

$\theta = \tan^{-1} 1$

$\theta = 45°$

$\boldsymbol{\theta_1 = 45°}$

$\theta_2 = 180° + 45° = \mathbf{225°}$



$*\theta \in \mathbf{45°, 225°}+$

Example 2: Solve $2cos\theta + \sqrt{3} = 0, 0° \leq \theta \leq 360°$

$2cos\theta + \sqrt{3} = 0$

$2cos\theta + \sqrt{3} - \sqrt{3} = 0 - \sqrt{3}$

$2cos\theta = -\sqrt{3}$

$\frac{2cos\theta}{2} = \frac{-\sqrt{3}}{2}$

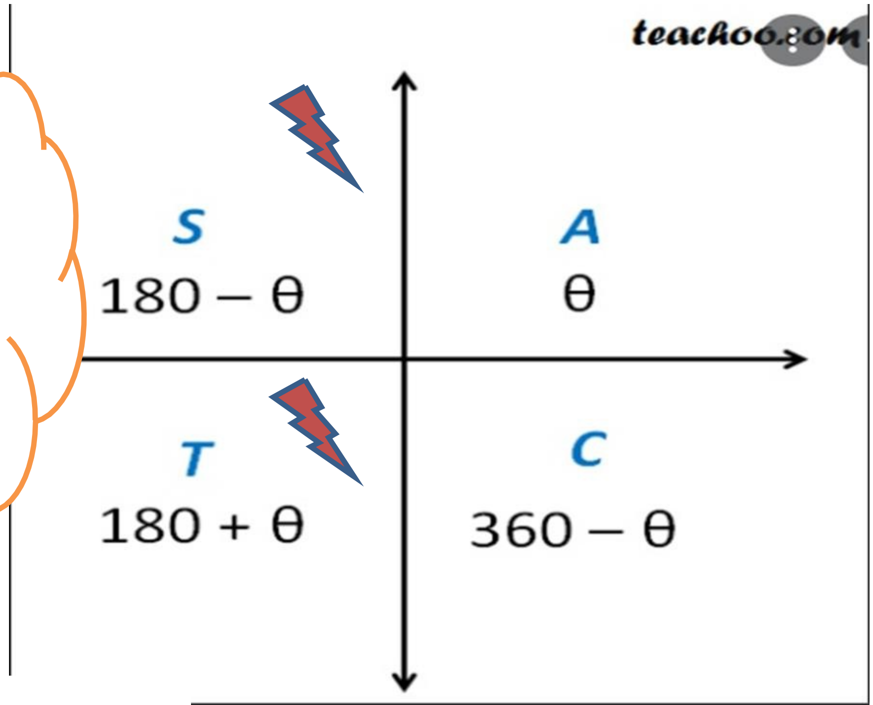
$cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = \cos^{-1}\frac{\sqrt{3}}{2}$

$\theta = 30°$

Since cos is negative

θ lies in quadrant II and III



$\theta_1 = 180° - 30°$

$\boldsymbol{\theta_1 = 150°}$

$\theta_2 = 180° + 30°$

$\theta_2 = 210°$

$\therefore \theta \in \{\mathbf{150°, 210°}\}$

Activity

2018:

4. Solve the **trigonometric equation** $\sqrt{3}\tan\theta = 1$ for $0° \leq \theta \leq 360°$ **(2 marks)**

2017:

4. Solve the **trigonometric equation** $\cos(\theta + 45°) = \frac{1}{2}$ for $0° \leq \theta \leq 360°$ **(2 marks)**

Strand	5 TRIGNOMETRY
Sub Strand	5.2.1explore trig identities and draw graphs
Content Learning Outcome	Students should be able to: • Sketch trigonometric graphs in the required domain.

Lesson notes: <u>Trigonometric graphs</u>

The general form of the Trigonometric function is $y = A \,{}^{sin}_{cos}(Bx \pm C) \pm k$ **where**

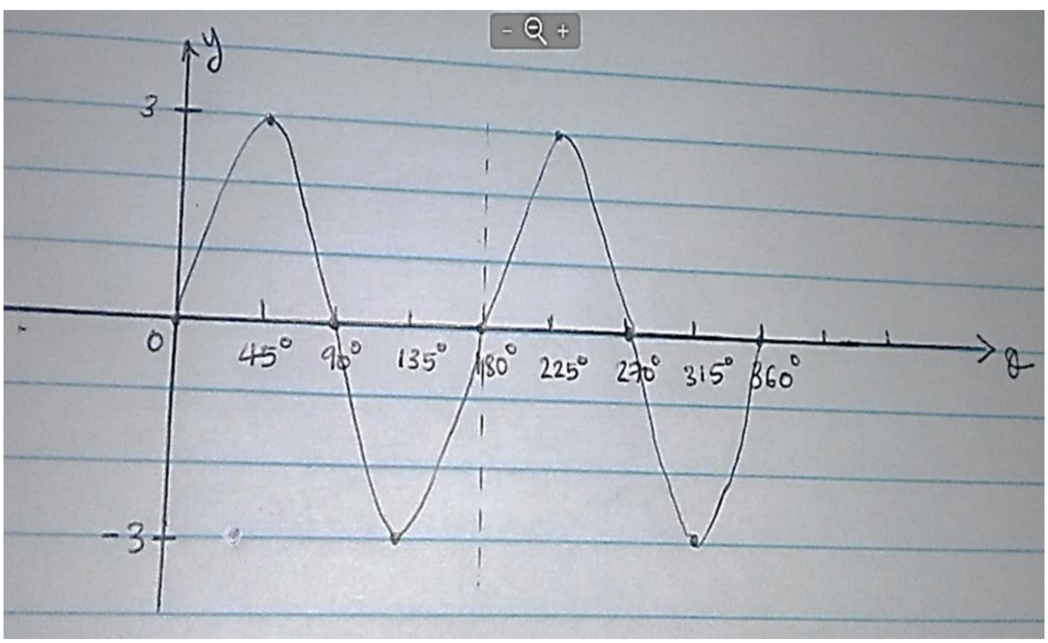
- A $\rightarrow$ is the amplitude of the graph. (height)
- B $\rightarrow$ 360/B is the period of the function. A complete turn of the curve from 0 to period.
  - ✓ $+\frac{C}{B}$: **Positive** $\frac{C}{B}$ the graph will shift towards left
  - ✓ $-\frac{C}{B}$: **Negative** $\frac{C}{B}$ the graph will shift towards right
- K $\rightarrow$ **Positive** k the graph will shift up
  : **Negative** k the graph will shift down

<u>Example 1</u> Sketch $y = 3\sin 2x \; for \; 0° \leq \theta \leq 360°$

$y = 3\sin 2x$

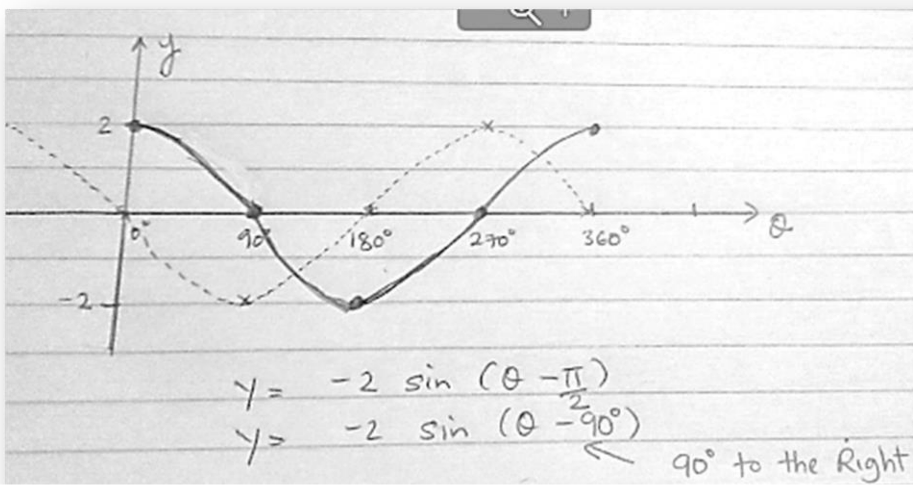
$A = 3$

$Period = \frac{360°}{2} = 180°$



Example 2: Sketch the graph of $y = -2\sin(\theta - \frac{\pi}{2})$ for $0° \leq \theta \leq 360°$

$y = -2\sin(\theta - 90°)$

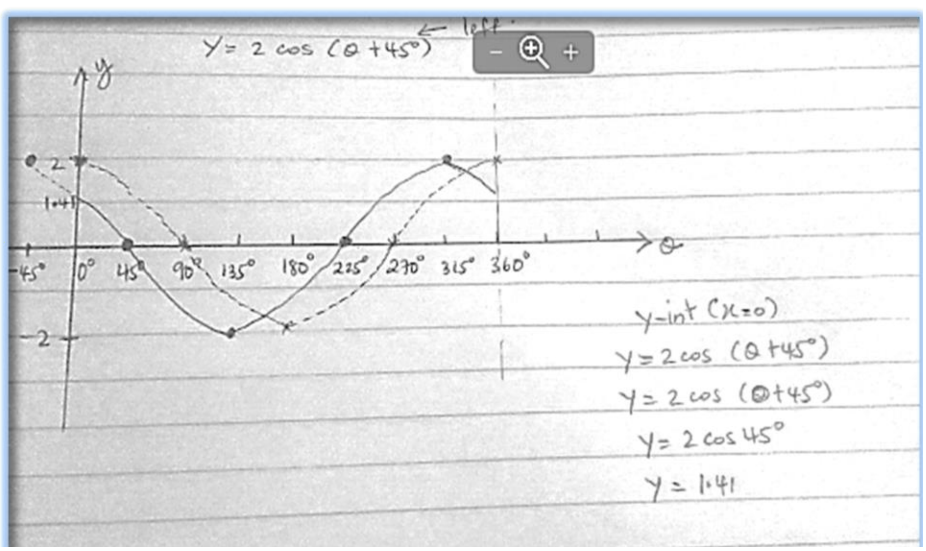


A= 2

Period $= \frac{360°}{1} = 360°$

Example 3: $y = 2\cos(\theta + \frac{\pi}{4})$ for $0° \leq \theta \leq 360°$

$y = 2\cos(\theta + 45°)$



A= 2

Period $= \frac{360°}{1} = 360°$

Strand	6 MATRIX
Sub Strand	6.1.1 EXPLORE AND STUDY OBJECTS USING MATRIX
Content Learning Outcome	Students should be able to: • Recall properties of matrix and find determinant and inverses of a 2 by 2 matrix. • Find scalar and matrix multiplication.

Lesson notes **Strand 6 Matrices And Transformation**

Recall : year 11

Determinant of a 2 by 2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = ad - bc$$

Inverse of a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\underbrace{ad-bc}_{\text{determinant}}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples

i) find the determinant of $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (1)(4) - (2)(3)$$

$$= 4 - 6$$

$$= -2 \checkmark$$

ii)

Finding Inverses 2x2

Example: Find the inverse of A.

$$A = \begin{bmatrix} 2 & 4 \\ -4 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(2)(-10)-(-4)(4)} \begin{bmatrix} -10 & -4 \\ 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} -10 & -4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & 1 \\ -1 & -\frac{1}{2} \end{bmatrix}$$

iii)

For example, given that $\mathbf{A} = \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix}$, let's find $2\mathbf{A}$.

To find $2\mathbf{A}$, simply multiply each matrix entry by 2:

$$2\mathbf{A} = 2 \cdot \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 10 & 2 \cdot 6 \\ 2 \cdot 4 & 2 \cdot 3 \end{bmatrix}$$

iv) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $and\, B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, find AB

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1(5) + 2(7) & 1(6) + 2(8) \\ 3(5) + 4(7) & 3(6) + 4(8) \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

<u>Activity</u>

1. Given matrix $A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

   a) Find $|A|$

   b) Find the inverse of A, A^{-1}.

   c) Evaluate $A.A^{-1}$

   d) Find $A.B$

2. Evaluate:

   a) $\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ -1 & -2 \end{pmatrix}$

   b) $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Activity

Exercise 45

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No. 1 and 4

Strand	6 MATRIX
Sub Strand	6.1.1 EXPLORE AND STUDY OBJECTS USING MATRIX
Content Learning Outcome	Students should be able to: • Calculate the image of the point after matrix transformation. • Identify the transformation reflection along the x and y axis.

Lesson notes **Transforming Points**

- The ordered pairs(x, y) can be represented by a 2 x 1 matrix:$\begin{pmatrix} x \\ y \end{pmatrix}$.
- To transform a point A(object) to image A'

✓ Write the transformation matrix first.
✓ Change the coordinates of the points to a 2 x 1 matrix and write beside the transformation matrix.
✓ Perform matrix multiplication to get the image point.
✓ Write image point as (x, y).

Note: A unit square has side of length 1 unit.

Examples

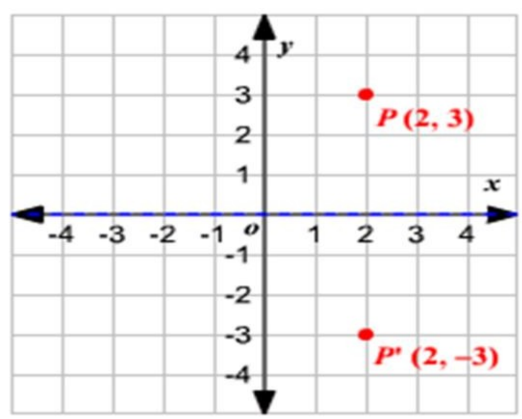
Consider point $P = (2,3)$. Transform point P using the Matrix $M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1(2) + 0(3) \\ 0(2) + -1(3) \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Therefore, $P = (2, -3)$



Reflection Matrices: Matrix multiplication can be used to reflect a figure.The general matrix for reflection are

Reflection over the:	Symbolized by:	Multiply the vertex matrix by:
x-axis	$R_{x\text{-axis}}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
y-axis	$R_{y\text{-axis}}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Examples

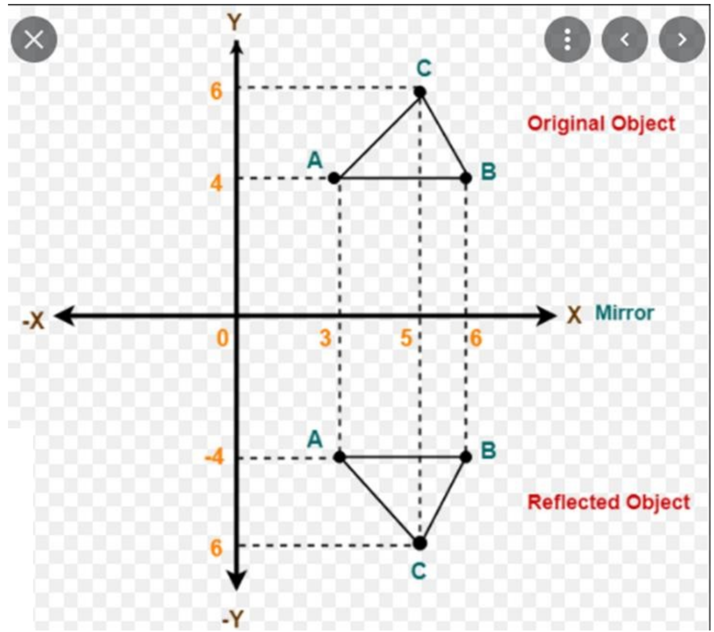
The diagram on the right shows ΔABC.

ΔABC is transformed by matrix $M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(i) Find the coordinates of A', B and $C'$the

images of A, B and C under the transformation matrix M.

(ii) Describe fully the transformation given by matrix M.



Solutions

(i)

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 3 & 6 & 5 \\ 4 & 4 & 6 \end{pmatrix}$

$= \begin{pmatrix} 1(3) + 0(4) & 1(6) + 0(4) & 1(5) + 0(6) \\ 0(3) + -1(4) & 0(6) + -1(4) & 0(5) + -1(6) \end{pmatrix}$

$= \begin{pmatrix} 3 & 6 & 5 \\ -4 & -4 & -6 \end{pmatrix}$

Therefore:

$A' = (3, -4)$

$B = (6, -4)$

$C' = (5, -6)$

(ii) Reflection along the x axis

Activity

Exercise 46

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No. 4,5 and 6