PENANG SANGAM HIGH SCHOOL

WORKSHEET BATCH SEVEN WEEK 19-21-MATHEMATICES YEAR12

Strand	7 Statistics
Sub Strand	7.1.1 Statistical Analysis
Content Learning Outcome	Students should be able to: Determine the measures of central tendency Work out the spread of a set of data

Lesson notes **Strand 7: Statistics**

Measures of central tendency

There are three main measures of central tendency, the **mode**, the **median** and the **mean**.

The **mode** is the **most common score** or the score that has the highest frequency.

The **median** is the **middle score** when the scores are listed in an ascending or descending order.

The **mean** is commonly called **average**, which is calculated using the formula:

Formula:

Ungrouped Data: $\bar{x} = \frac{x_1 + x_2 + \cdots x_n}{n}$

Grouped Data: $\bar{x} = \frac{\Sigma f x}{n}$

Where: f = frequency in each class

x = midpoint of each class

n = total number of scores

Adding a constant to each score

If a constant c is added to each score then the **new mean will be:** $x + c$ and the standard deviation will remain unchanged.

Multiplying a constant to each score

If a constant c is multiplied to each score then the new mean will be: $c x$ and the new standard

Finding the spread

Measures of spread include: range, quartiles, interquartile range and standard deviation.

Range: highest score –lowest score

Interquartile range: upper quartile – lower quartile

Standard deviation:

ungrouped data:	*grouped or frequency tables:*

Examples

1. For the following list of scores find the mean, median, mode, range, interquartile range and standard deviation.

$$5,6,10,4,7,2,8,8,5$$

Solutions:

$$2,4,5,5,6,7,8,8,10$$

Mean: $\frac{2+4+5+5+6+7+8+8+10}{9}=\frac{55}{9}=6.11$

Standard deviation:

Median: **6**

Mode: **5 and 8**

Range: **HS-LS**

$=10 \quad 2=8$

$=2.28$

Interquartile range: $UQ \quad LQ$

$8 \quad 4.5=3.5$

Example 2 Calculate the standard deviation for the following data.

x	f
1	1
2	4
3	9
4	6
total	20

Mean=

x	f	fx		
1	1	1	$(1 \quad 3)^2=4$	4
2	4	8	$(2 \quad 3)^2=1$	4
3	9	27	$(3 \quad 3)^2=0$	0
4	6	24	$(4 \quad 3)^2=1$	6

Activity: Exercise 49 Page:196 numbers 3 and 4

Str	
Sub Strand	8.1.1Probability experiments
Content Learning Outcome	Students should be able to: Calculate probabilities with replacement. Use probability tree.

Lesson notes **<u>Strand 8:Probabilty</u>**

Probability of any event = P(E) = $\frac{\textit{successful out come}}{\textit{All possible out come}}$ = $\frac{\textit{no. of times E occurs}}{\textit{Total possible outcomes}}$

Note: 0 P(E) 1

<u>SAMPLE SPACE</u>
All possible outcomes denoted by (S)

<u>EQUALLY LIKELY OUTCOMES</u>
Are outcomes that have the same chance of occurring

<u>HOW TO FIND THE SAMPLE SPACE</u>
1 experiment: look for all the out comes in the object.
2 experiment or 1 experiment with 2 object use x/y axis or use tree diagram. Label the branches.
3 experiments: The tree diagram

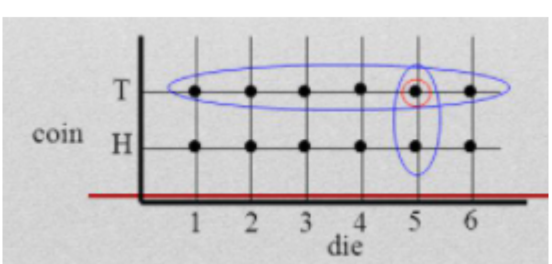
<u>EXAMPLE</u>
1) A die is rolled. Find the probability of obtaining:
I) Number 3 II) Number 5 III) Even number IV) Multiples of 4 V) Factors of 6

ANSWER:
Sample space = {1,2,3,4,5,6}
(i).P (3) =1/6 (ii) P (5) =1/6 (iii)P (E.N) =3/6 =1/2 (iv) P (M.4) =1/6
(v) P (F.6) =4/6=2/3

2) A coin and a die is rolled simultaneously, find the probability of getting:
i) A even no and head. ii) Less then 3 and tail

Answer:
S= { (H,1), (H ,2), (H,3), (H ,4), (H,5), (H ,6), (T,1), (T ,2), (T,3),(T,4), (T,5), (T ,6)}
i. P(even no and H) = 3/12 = ii P (<3 and tail) =2/12 = 1/6

SAN



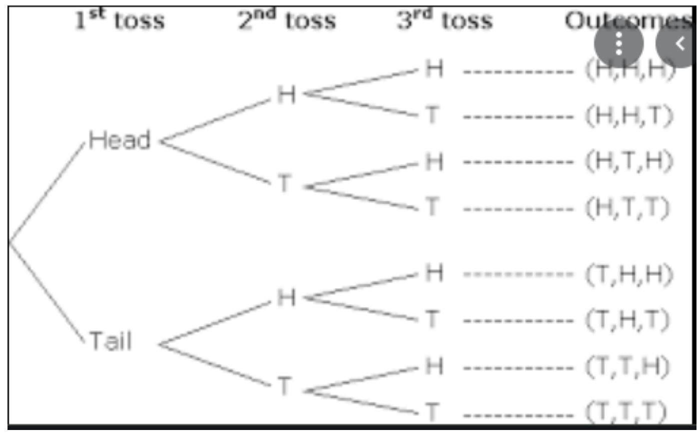
3) A coin is tossed 3 times

i) Find the probability of getting all head. ii) At least 2 tail.

Sol: S = { (HHH), (HHT), (HTT), (HTH), (THT), (THH), (TTH), (TTT)}

i. P(HHH)=1/8

ii P(at least 2 tail)
=
P(HTT)+P(THT)+P(TTH)+P(TTT)
=



PROBABILITY WITH REPLACEMENT

1) A bag contains 3 red marbles and 2 yellow marbles. One marble is randomly picked and its colour is noted. It is replaced in the bag and the second one is withdrawn and its colour is noted.

i) Find the sample space.

ii) Find the probability of obtaining:

a) 2 red marbles b) Marbles of same colour. c) Marbles of different colours.

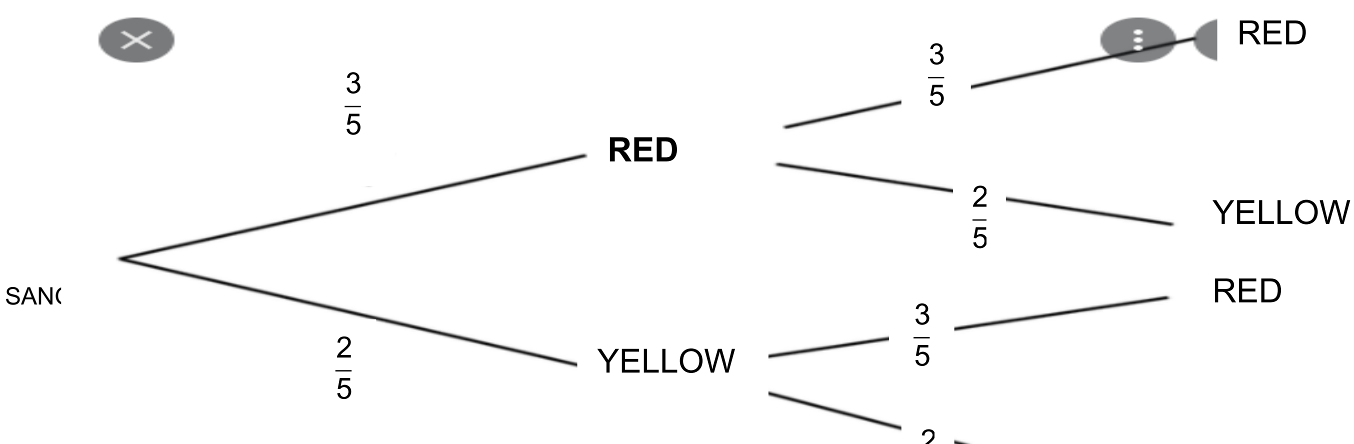
Solutions

i) S = { (R_1, R_1), (R_1, R_2), (R_1, R_3), (R_2, R_1), (R_2, R_2), (R_2, R_3), (R_3, R_1), (R_3, R_2), (R_3, R_3), (Y_1, R_1), (Y_1, R_2), (Y_1, R_3), (Y_1, Y_1), (Y_1, Y_2), (Y_2, R1), (Y_2, R_2), (Y_2, R_3), (Y_2, Y_1), (Y_2, Y_2), (R_1, Y_1), (R_1, Y_2), (R_2, Y_1), (R_2, Y_2), (R_3, Y_1), (R_3, Y_2),}

ii) a. P (2R) = RR = 9/ 25

b. P (same colour) = RR+ YY= 13/25

c. P (different colour) = 1 – P(same colour) or RY + YR = 12/25



Strand	8 Probability
Sub Strand	8.1.1Probability experiments
Content Learning Outcome	Students should be able to: Calculate probabilities with replacement. Use probability tree.

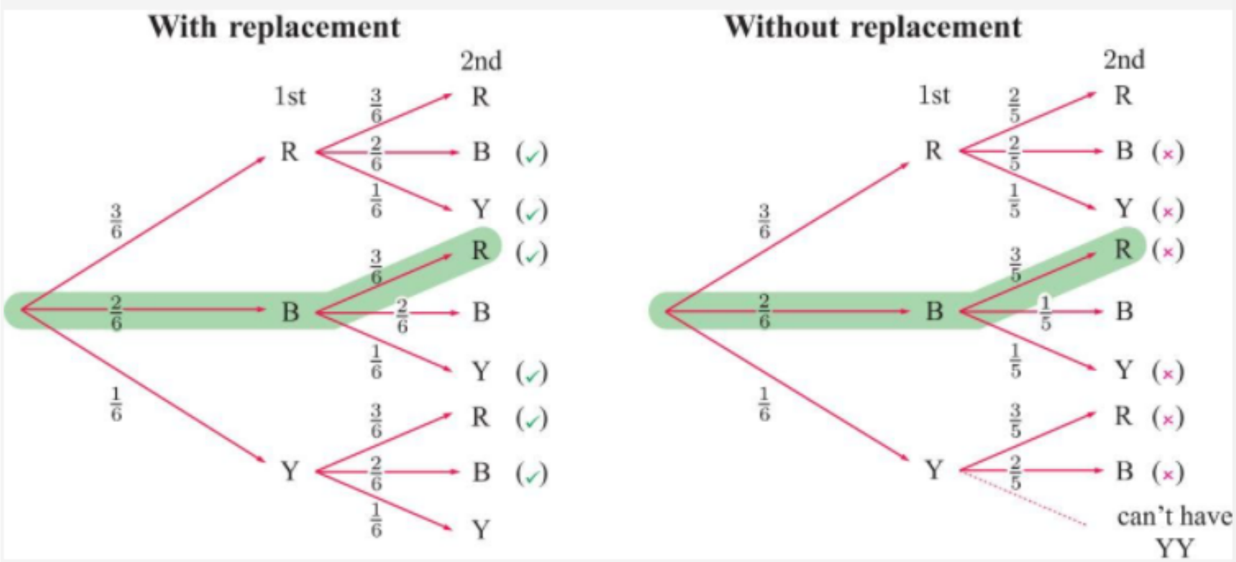
Lesson notes **Strand 8:Probabilty**

PROBABILITY WITHOUT REPLACEMENT

Consider a box containing 3 red, 2 blue, and 1 yellow marble. If we sample two marbles, we can do this either:

- **with replacement** of the first before the second is drawn, or
- **without replacement** of the first before the second is drawn.

Examine how the tree diagrams differ:



Example

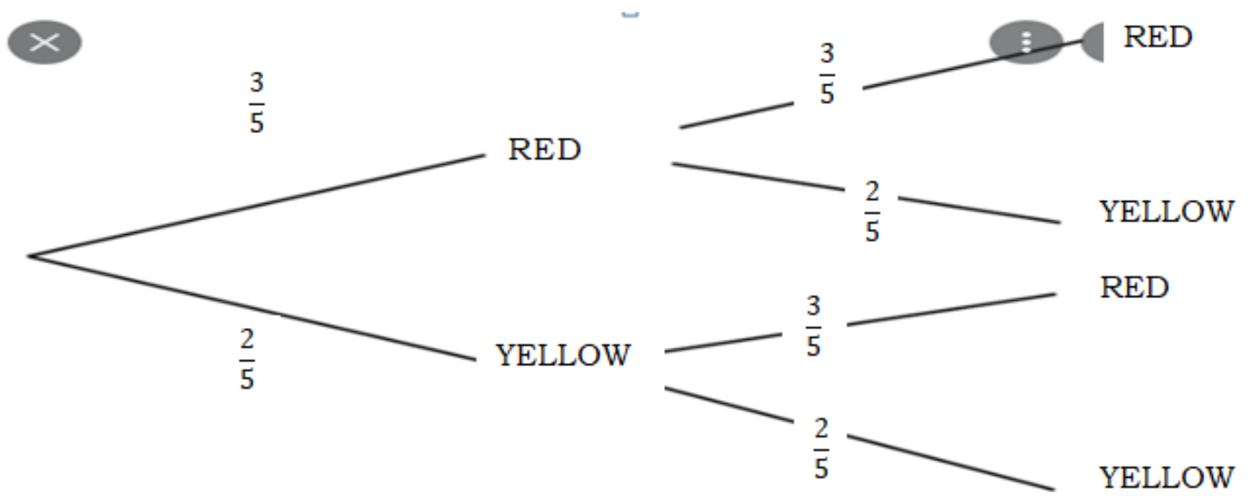
1) A bag contains 3 red marbles and 2 yellow marbles. One marble is randomly picked and its colour is noted. It **is not replaced** in the bag and the second one is withdrawn and its colour is noted.

i) Find the sample space.

ii) Find the probability of obtaining:

a) 2 red marbles b) Marbles of same colour. c) Marbles of different colours.

S= { (R_1, R_2), (R_1, R_3), (R_2, R_1), (R_2, R_3), (R_3, R_1), (R_3, R_2), (Y_1, R_1), (Y_1, R_2), (Y_1, R_3), (Y_1, Y_2), $(Y_2, R1)$, (Y_2, R_2), (Y_2, R_3), (Y_2, Y_1), , (R_1, Y_1), (R_1, Y_2), (R_2, Y_1), (R_2, Y_2), (R_3, Y_1), (R_3, Y_2),}



a. $P(\ 2R) = RR\ = 6/\ 20 = \frac{3}{10}$

b. $P(\textit{same colour}) = RR + YY = \frac{8}{20} = \frac{2}{5}$

c. $P(\textit{different colour}) = 1 - P(\textit{same colour})\ or\ \ RY\ + YR\ = \frac{12}{20} = \frac{3}{5}$

<u>Exercise</u>

1.A fair six-sided die has 3 faces painted red, two faces painted yellow and one face painted blue.

If the die is tossed twice :

(i) calculate the probability that both tosses give the same colour.

(ii) calculate the probability that the colours are different.

2. A box contains two green, one blue and two yellow balls, all of the same shape and size. A ball is picked at random from the box.
What is the probability that the ball picked will be:
(i) green? (ii) not yellow?

If the first ball is picked and not replaced and a second one is picked from the box, what is the probability that:
(iii) the first ball picked is blue and the second is yellow?
(iv) both balls are of the same colour?

Content Learning Outcome	Students should be able to: Calculate probabilities using venn diagrams. Find the expected number.

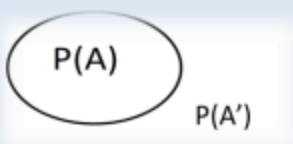
Lesson notes **<u>Strand 8 :Probabilty</u>**

<u>Probabilities Using Venn Diagram</u>

Note: A **Venn diagram** is constructed with a collection of simple closed curves drawn in a plane as well as overlapping circles.

Properties of Venn diagram:

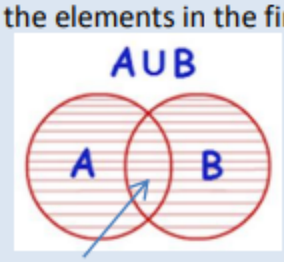
- The interior of the circle symbolically represents the elements of the set (A) while the exterior represents elements that are not members of the set (A' or A^c). This is also referred to as **Compliment of A**.



P(A) + P(A') = 1

$P(A) = 1 - P(A^c)$
$P(A^c) = 1 - P(A)$

- **Empty Set** is a set with no elements common. Shown by { } or Ø.
  A$\cap$ $\boldsymbol{B} = \emptyset$ ***or*** { } **null set**
- Two sets, A and B, is said to be **disjoint** if **A**$\cap$ $\boldsymbol{B} = \emptyset$
- "**Union**" of sets has the special symbol ∪ that means it is the set containing all the elements in the first or in the second set.



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

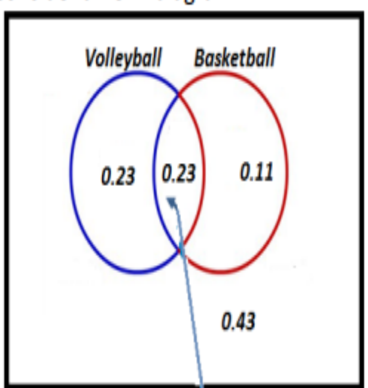
- $P(A \cap B)$: "Intersection" of a set has special symbol ∩ and it must be in both sets, A and B.

Example 1 In a school, the probability that a student can play volleyball is 0.46 and the probability that a student can play basketball is 0.34. The probability that the student can play **both** is 0.23.
Find the probability that a randomly selected student can play:

a) Volleyball only.
b) volleyball or basketball.
c) Neither volleyball nor basketball.

Solutions

Consider a Venn diagram:



play Both (Volleyball and Basketball

Total Probability to be one so subtract from **one** This is the compliment so 'neither nor'

a) $P(\textit{volleyball only}) = 0.23$

b) $P(VUB) = P(V) + P(B) \quad P(V \quad B)$

$= 0.46 + 0.34 \quad 0.23$

$= 0.57$

c) $P(\textit{neither V or B}) = 0.43$

Example 2: Two sets, A and B, is **disjoint.** If P(A) = 0.4 P(B) = 0.3,

a) what is the probability of A and B occurring?
b) what is the probability of A **or** B occurring?

a) The word refers to P(A∩B):

$$P(A \cap B) = 0$$

b) The word **or** refers to P(AUB):

$$P(A\,UB) = P(A) + P(B) - P(A \cap B)$$
$$= 0.4 + 0.3 - 0$$
$$= 0.7$$

Expected Number

Note: Recall that probability is the chance that an event will happen. We can use this probability to predict the number of times an event will happen in the future. This in known as Expected value. To find the Expected number, multiply the probability of the outcome by the number of trials.

$$\boxed{E = n \times p}$$

where
E – Expected number
n – Number of trials
p – Probability of event

Content Learning Outcome	Students should be able to: State characteristics of normal distribution Define normal distribution terms.

Lesson notes **Strand 8: Probability**

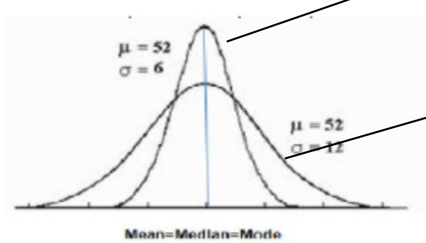
Characteristics of Normal distribution

- Normal probability distribution characteristics

✓The normal curve is bell-shaped.

✓The normal distribution is symmetric about its mean(50% of all values lie either site of the mean).

✓N al

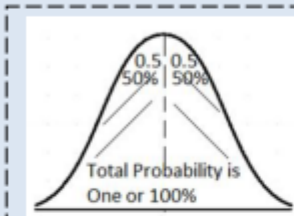


When the standard deviation is small, the curve is tall and narrow.

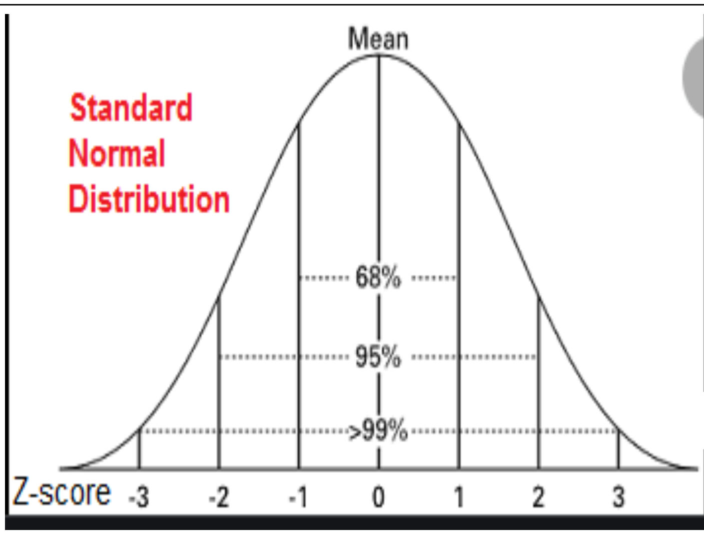
When the standard deviation is large, the curve is short and wide.

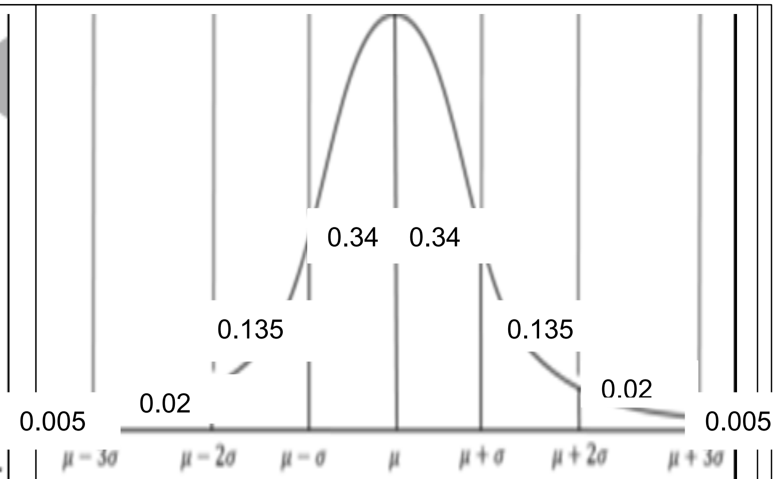
Normal distribution terms

Note: The normal distribution is a continuous probability distribution. This has several implications for probability.



- The total **area under** the normal curve is equal to **one**.
- A score is
  - **probable** or **likely** to lie in this range : $\mu - 1\sigma < x < \mu + 1\sigma$
  - **very likely or very probably** to lie in this range : $\mu - 2\sigma < x < \mu + 2\sigma$
  - **almost certain** to lie in this range : $\mu - 3\sigma < x < \mu + 3\sigma$





Example Your Company packages sugar in one kg bags. When you weigh a sample of bags you get these results:

1007g, 1032g, 1002g, 983g, 1004g, 1040g, 1021g, 999g, 1009g, x g

a) Find the mean weight of 9 bags of sugar.
b) Another weight x g is to be included such that the mean weight is 1010g. Find the weight of the 10th bag.
c) Work out the standard deviation.
d) Assume that the weight of the sugar is normally distributed with a mean of 1010g standard deviation of 16g. Draw the normal distribution curve.
e) Between what weights is the bag likely to be?
f) Between what weights is the bag almost certain to be?

Solutions

a) $$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n} = \frac{1007 + 1032 + \ldots\ldots + 1009}{9} = \frac{9097}{9} = 1010\frac{7}{9}\,g$$

c) Put calculator mode on SD. Press each number with M+ sign.

Press ☞ 100 | M+ | 1032 | M+ | 100 | M+ | 98 | M+

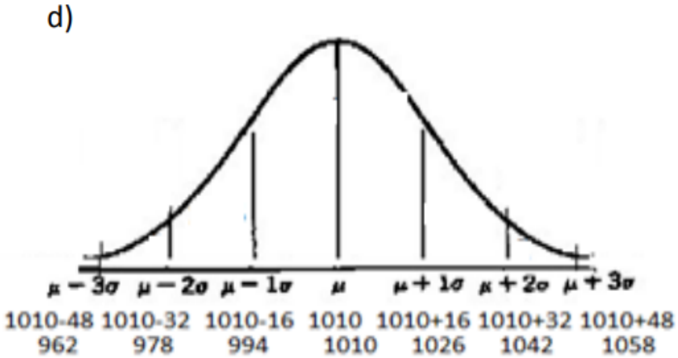
☞ 100 | M+ | 104 | M+ | 102 | M+ | 99 | M+

☞ 100 | M+ | 100 | M+

Shift | 2 | 2 | =

$\sigma = 15.86 \approx 16g$

d)



e) **likely** to be:

$\mu - 1\sigma < x < \mu + 1\sigma$

$994g < x < 1026g$

f) **almost certainly**:

$\mu - 3\sigma < x < \mu + 3\sigma$

$962g < x < 1058g$

<u>Activity</u>

1. A brand of batteries have a mean lifetime of 400hours with a standard deviation of 50 hours. If a battery is selected at random, what is the probability that:

a. It has a lifetime of more than 400 hours.

b. It has a lifetime between 350 hours and 500 hours.

c. It has a lifetime of less than 300 hours.

D. From a sample of 200 batteries, how many do you expect to be with a lifetime of less than 300 hours.