3055 BA SANGAM COLLEGE

PH: 6674003/9264117 E-mail: basangam@connect.com.fj

Worksheet 19

School: Ba Sangam College **Year / Level: 13**
Subject: Mathematics **Name of student: _____**

Strand	7 – Probability and Inferential Statistics
Sub strand	7.3 – Binomial Distribution
Content Learning Outcome	Recognize situations where binomial distribution is a suitable model and use this to solve problems

Binomial Distribution

Ref. Yr 13 Mathematics Textbook *pg. 163 – 167*

Binomial Probabilities

The binomial distribution has **two possible outcomes** (the prefix "bi" means two).

Example: A coin has only two possible outcomes: heads or tails and taking a test has two possible outcomes: pass or fail.
The two outcomes are called **success or failure.**

- Properties of Binomial Experiment:
  1. The experiment consists of 'n' repeated trials
  2. Only two possible outcomes (Success or Failure)
  3. Probability of success is the same for each trial.
  4. Each trial is independent of each other

- The binomial distribution formula is:

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \quad , x = 0, 1, 2, 3, \ldots\ldots\ldots n$$

where:

P = binomial probability
x = total number of "successes"
p = probability of success
q = probability of failure ($q = 1 - p$)
n = number of trials

Example 1 If 20% of the bulbs produced by a factory are faulty, determine the probability that out of sample of 12 randomly chosen bulbs, one will be faulty.

Answer

Given : $n = 12$, $p = 20\% = 0.20$, $q = 1 - 0.20 = 0.80$,
$x = 1$ (one will be faulty)

$$\text{Probability} = \binom{n}{x} p^x q^{n-x}$$

$$= \binom{12}{1} (0.2)^1 (0.8)^{12-1}$$

$$= 0.2062$$

Table of Binomial probabilities for Individual terms

BINOMIAL DISTRIBUTION; INDIVIDUAL TERMS

Tabulated values are $P(X = x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$ for certain values of n, π. If $\pi > .50$ use $P(Y = y) = \binom{n}{y} \pi_1^y (1 - \pi_1)^{n-y}$ where $\pi_1 = 1 - \pi$, $y = n - x$.

n	x	π = .01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
2	0	.9801	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500
	1	.0198	.0950	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000
	2	.0001	.0025	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
3	0	.9703	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250
	1	.0294	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750
	2	.0003	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750
	3		.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250
4	0	.9606	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625
	1	.0388	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500
	2	.0006	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750
	3		.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500
	4			.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625
5	0	.9510	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313
	1	.0480	.2036	.3281	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563
	2	.0010	.0214	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125
	3		.0011	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125
	4			.0004	.0022	.0064	.0146	.0283	.0488	.0768	.1128	.1563
	5				.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313
6	0	.9415	.7351	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277	.0156
	1	.0571	.2321	.3543	.3993	.3932	.3560	.3025	.2437	.1866	.1359	.0938
	2	.0014	.0305	.0984	.1762	.2458	.2966	.3241	.3280	.3110	.2780	.2344
	3		.0021	.0146	.0415	.0819	.1318	.1852	.2355	.2765	.3032	.3125
	4		.0001	.0012	.0055	.0154	.0330	.0595	.0951	.1382	.1861	.2344
	5			.0001	.0004	.0015	.0044	.0102	.0205	.0369	.0609	.0938
	6					.0001	.0002	.0007	.0018	.0041	.0083	.0156
7	0	.9321	.6983	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152	.0078
	1	.0659	.2573	.3720	.3960	.3670	.3115	.2471	.1848	.1306	.0872	.0547
	2	.0020	.0406	.1240	.2097	.2753	.3115	.3177	.2985	.2613	.2140	.1641
	3		.0036	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734
	4		.0002	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734
	5			.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641
	6				.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547
	7						.0001	.0002	.0006	.0016	.0037	.0078
8	0	.9227	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039
	1	.0746	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0313
	2	.0026	.0515	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094
	3	.0001	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188
	4		.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734
	5			.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188
	6				.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094
	7					.0001	.0004	.0012	.0033	.0079	.0164	.0313
	8							.0001	.0002	.0007	.0017	.0039
9	0	.9135	.6302	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.0830	.2985	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.0034	.0629	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0001	.0077	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4		.0006	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5			.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6			.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7					.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8						.0001	.0004	.0013	.0035	.0083	.0176
	9								.0001	.0003	.0008	.0020
10	0	.9044	.5987	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.0914	.3151	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.0042	.0746	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0001	.0105	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4		.0010	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5		.0001	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6			.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7				.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172

BINOMIAL DISTRIBUTION. INDIVIDUAL TERMS

n	v	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
10	8					.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9							.0001	.0005	.0016	.0042	.0098
	10									.0001	.0003	.0010
11	0	.8953	.5688	.3138	.1673	.0859	.0422	.0198	.0088	.0036	.0014	.0005
	1	.0995	.3293	.3835	.3248	.2362	.1549	.0932	.0518	.0266	.0125	.0054
	2	.0050	.0867	.2131	.2866	.2953	.2581	.1998	.1395	.0887	.0513	.0269
	3	.0002	.0137	.0710	.1517	.2215	.2581	.2568	.2254	.1774	.1259	.0806
	4		.0014	.0158	.0536	.1107	.1721	.2201	.2428	.2365	.2060	.1611
	5		.0001	.0025	.0132	.0388	.0803	.1321	.1830	.2207	.2360	.2256
	6			.0003	.0023	.0097	.0268	.0566	.0985	.1471	.1931	.2256
	7				.0003	.0017	.0064	.0173	.0379	.0701	.1128	.1611
	8					.0002	.0011	.0037	.0102	.0234	.0462	.0806
	9						.0001	.0005	.0018	.0052	.0126	.0269
	10								.0002	.0007	.0021	.0054
	11										.0002	.0005
12	0	.8864	.5404	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.1074	.3413	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.0060	.0988	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0002	.0173	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4		.0021	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5		.0002	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6			.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7				.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8				.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9					.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10							.0002	.0008	.0025	.0068	.0161
	11								.0001	.0003	.0010	.0029
	12										.0001	.0002
15	0	.8601	.4633	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	
	1	.1303	.3658	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005
	2	.0092	.1348	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032
	3	.0004	.0307	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139
	4		.0049	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417
	5		.0006	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916
	6			.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527
	7			.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964
	8				.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964
	9				.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527
	10					.0001	.0007	.0030	.0096	.0245	.0515	.0916
	11						.0001	.0006	.0024	.0074	.0191	.0417
	12							.0001	.0004	.0016	.0052	.0139
	13								.0001	.0003	.0010	.0032
	14										.0001	.0005
	15											
20	0	.8179	.3585	.1216	.0388	.0115	.0032	.0008	.0002			
	1	.1652	.3774	.2702	.1368	.0576	.0211	.0068	.0020	.0003	.0001	
	2	.0159	.1887	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002
	3	.0010	.0596	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011
	4		.0133	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046
	5		.0022	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148
	6		.0003	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370
	7			.0020	.0160	.0545	.1124	.1643	.1844	.1659	.1221	.0739
	8			.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201
	9			.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602
	10				.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762
	11					.0005	.0030	.0120	.0336	.0710	.1185	.1602
	12					.0001	.0008	.0039	.0136	.0355	.0727	.1201
	13						.0002	.0010	.0045	.0146	.0366	.0739
	14							.0002	.0012	.0049	.0150	.0370
	15								.0003	.0013	.0049	.0148
	16									.0003	.0013	.0046
	17										.0002	.0011
	18											.0002
	19											
	20											

Example 1 For the previous example, use the table to find the probability.

Answer

Given : $n = 12$, p or $\pi = 20\% = 0.20$, $q = 1 - 0.20 = 0.80$, $x = 1$

n	x	π 0.20
12	0	↓
	1	→ 0.2062
	.	
	.	
	.	
	11	
	12	

∴ Probability = 0.2062

Example 2 If 20% of the bulbs produced by a factory are faulty, determine the probability that out of sample of 12 randomly chosen bulbs, at least 8 will be faulty.

Answer

Given : n = 12, p = 20% = 0.20, x = (8, 9, 10, 11, 12)
Reading Probability from the table of individual terms and add up the probability values corresponding to $x = 8$ to $x = 12$.

n	x	π 0.20
12	.	.
	.	.
	.	.
	8	→ 0.0005
	9	→ 0.0001
	10	
	11	
	12	

Probability = 0.0005 + 0.0001

= 0.0006

ACTIVITY

1.
A maths teacher sets up study groups in her maths class. Each study group has 3 students. If 20 % of the maths students in her class are females, what is the probability that at least one member of a group is a female?

(2 marks)

2.

A survey in a country shows that 95% of the people love listening to music. What is the probability that from 12 people interviewed on the streets, at least 11 will be found to have love for music?

(2 marks)

3.

A survey on the streets of Suva on a sunny day showed that 80% of the people wore sunglasses. What is the probability that exactly 10 out of 15 people will be wearing sunglasses on a particular sunny day?

(2 marks)

THE END