PENANG SANGAM HIGH SCHOOL
P.O.BOX 44, RAKIRAKI
LESSON NOTES – WEEK 19 - 21

Year/Level: 13 **Subject:** Mathematics

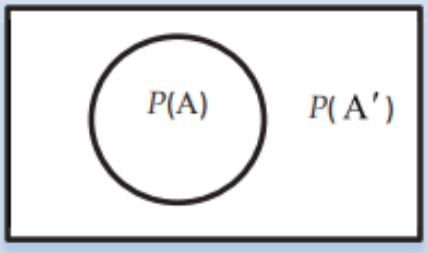
Strand	7 **PROBABILITY AND INFERENTIAL STATISTICS**
Sub Strand	7.1.1 PROBABILITY
Content Learning Outcome	Students should be able to; - Identify types of events - Calculate probabilities associated with these events.

Types of Events

Complementary Events

- The complement of event A (symbol A') means every outcome which is not in event A.
  - Example If event A is getting a pass in a test, then A' is getting a fail in a test.
- Two events are complementary if their probabilities add up to one.

$$P(A) + P(A') = 1$$



EXAMPLE:

1. The probability for getting a blue marble is 0.2 whereby $P(A) = 0.2$.
   Find $P(A')$

$P(A) + P(A') = 1$

$P(A') = 1 \quad P(A)$

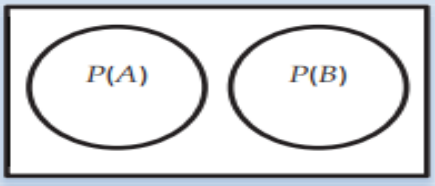
$= 1 \quad 0.2$

Exercise:

If $P(A) = 0.2$, $P(B) = 0.4$. Find $P(B')$

MUTUALLY EXCLUSIVE EVENTS

Mutually Exclusive Events have no outcomes in common.



- Intersection
  Since no outcome is common, then there will not be any intersection, i.e.

$A \cap B = \emptyset$ or $\{\}$ null set	$P(A \cap B) = 0$

- Union

$$P(A \cup B) = P(A) + P(B)$$

EXAMPLE 1:

Given $P(A) = 0.20$ and P (B) $= 0.41$. Find $P(A \cup B)$ given A and B are mutually exclusive?

$P(A \cup B) = P(A) + P(B)$

$= 0.20 + 0.41$

$= 0.61$

Example 2:

An experiment involves rolling two fair die. Events A and B are defined as

Event A: Getting a sum of 8

Event B: Getting a "1" on the first dice.

Are events A and B mutually exclusive? Explain

$= \frac{5}{36}$

$= \frac{6}{36} = \frac{1}{6}$

Yes, it is mutually exclusive because

$= 0$

1,1	1,2,	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Exercise:

1. An experiment involves rolling two fair die. Events A and B are defined as

Event A: Getting a sum of 6

Event B: Getting a double

Are events A and B mutually exclusive? Explain

2. $P(A) = 0.22$, $P(B) = 0.54$. . Find $P(AUB)$ given A and B are mutually exclusive?

$P(AUB) = P(A) + P(B)$

$= 0.22 + 0.54$

$= 0.76$

SANC

Strand	7 **PROBABILITY AND INFERENTIAL STATISTICS**
Sub Strand	7.1.1 PROBABILITY
Content Learning Outcome	Students should be able to; - Identify types of events - Calculate probabilities associated with these events.

INDEPENDENT EVENTS

Events A and B are independent if their occurrence do not affect each other.
$\Rightarrow$*Can happen together*

- Probability of A and B occurring together

$$P(A \cap B) = P(A).\ P(B)$$

Example:

Given $P(A) = 0.2$, $P(B) = 0.5$ and $P(A \quad B) = 0.10$. Are events A and B independent?

$P(A \quad B) = P(A).\ P(B)$

$= 0.2 \quad 0.5$

$= 0.1$

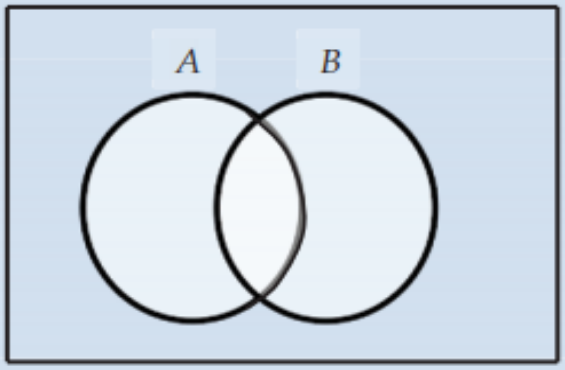
Exercise:

1. The probability that Jone passes Accounting (Event A) is 0.2 and the probability that Jone passes Economics (Event B) is 0.3. The probability that he passes both is 0.006.
   a. Are events A and B independent?

2. The probability that event A occurs is 0.60 while the probability that event B occurs is 0.25. The probability that both A and B occur is 0.12. Explain why events A and B are not independent.

Addition Rule

If A and B are not disjoint then



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example:

$P(A) = 0.3$, $P(B) = 0.4$ and $P(A \quad B) = 0.06$. Find:

a. $p(A')$

$P(A') = 1 \quad P(A)$

$= 1 \quad 0.3$

$= 0.7$

b. $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) \quad P(A \quad B)$

$= 0.3 + 0.4 \quad 0.06$

$= 0.7 \quad 0.06$

$= 0.64$

Exercise:

1. The Fiji National Bank has two computers. The probability that Computer A will break down once in a month is 0.05. The probability that Computer B will breakdown once in a month is 0.1. In a month and assuming that the results are independent.

a. What is the probability that either Computer A and B will break down?

b. What is the probability that neither Computer A nor computer B will break down?

2. The probability that Epeli passes Economics (Event E) is 0.2 and the probability that he passes Accounting (Event A) is 0.3. The probability that he passes both subjects is 0.06.
   a. What is the probability that Epeli passes either Economics and Accounting?
   b. What is the probability that Epeli fails both subjects?

3. The probability that Peter fails Maths (Event A) is 0.26 and the probability that he fails English (Event B) is 0.35. The probability that he fails both English and Maths is 0.18.
   a. Represent this information on a Venn Diagram.

b. Find the probability that Peter fails only Maths

c. What is the probability that Peter passes both subjects?

Strand	7 **PROBABILITY AND INFERENTIAL STATISTICS**
Sub Strand	7. 2. 1 Inverse Normal Distribution
Content Learning Outcome	Students should be able to; - Solve applied inverse normal distribution problems

Inverse Normal Distribution

➢ Table to be used:

INVERSE NORMAL DISTRIBUTION

For a listed value of P, the table gives the value of z such that the standardised normal variate Z has a probability P of lying between 0 and z. e.g. P = 0.30, z = .8416 means P(0 < Z < 0.8416) = .30

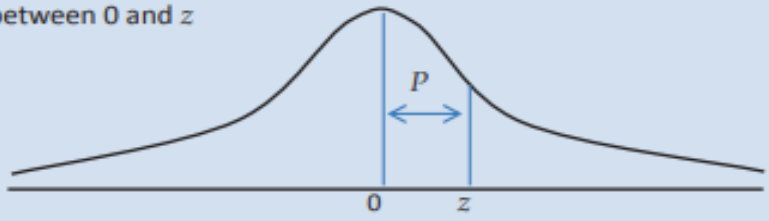
P	z	P	z	P	z	P	z	P	z	P	z
.00	.0000	.10	.2533	.20	.5244	.30	.8416	.40	1.2816	.470	1.8808
.01	.0251	.11	.2793	.21	.5534	.31	.8779	.41	1.3408	.471	1.8957
.02	.0502	.12	.3055	.22	.5828	.32	.9154	.42	1.4051	.472	1.9110
.03	.0753	.13	.3319	.23	.6128	.33	.9542	.43	1.4758	.473	1.9268
.04	.1004	.14	.3585	.24	.6433	.34	.9945	.44	1.5548	.474	1.9431
.05	.1257	.15	.3853	.25	.6745	.35	1.0364	.45	1.6449	.475	1.9600
.06	.1510	.16	.4125	.26	.7063	.36	1.0803	.46	1.7507	.476	1.9774
.07	.1764	.17	.4399	.27	.7388	.37	1.1264			.477	1.9954
.08	.2019	.18	.4677	.28	.7722	.38	1.1750			.478	2.0141
.09	.2275	.19	.4959	.29	.8064	.39	1.2265			.479	2.0335

P	z	P	z	P	z	P	z	P	z
.480	2.0537	.485	2.1701	.490	2.3263	.495	2.5758	.4995	3.2905
.481	2.0749	.486	2.1973	.491	2.3656	.496	2.6521	.4999	3.7190
.482	2.0969	.487	2.2262	.492	2.4089	.497	2.7478	.49995	3.8906
.483	2.1201	.488	2.2571	.493	2.4573	.498	2.8782	.49999	4.2649
.484	2.1444	.489	2.2904	.494	2.5121	.499	3.0902		

➢ Conversion formula

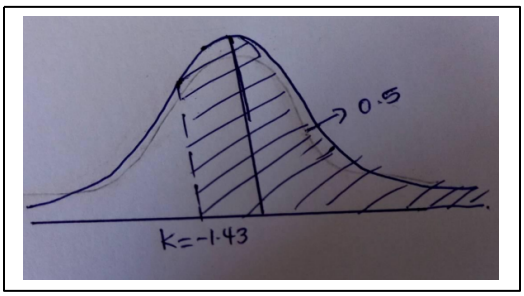
$$Z = \frac{x - \mu}{\sigma}$$

➢ P is between 0 and z



Examples:

1. **Find k given that** $P(z>k)=0.9236$

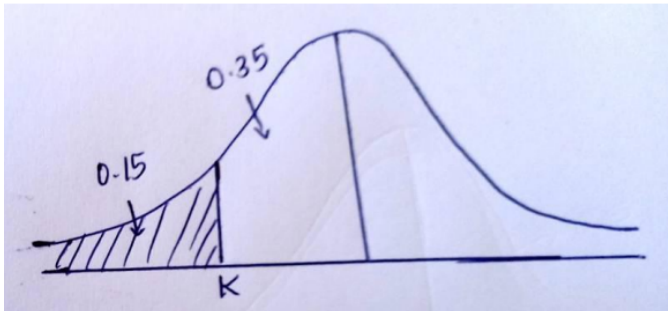


$k=$ 1.43

0.9236 0.5 = 0.4236

z	0	1	2	3	4	5	6
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279

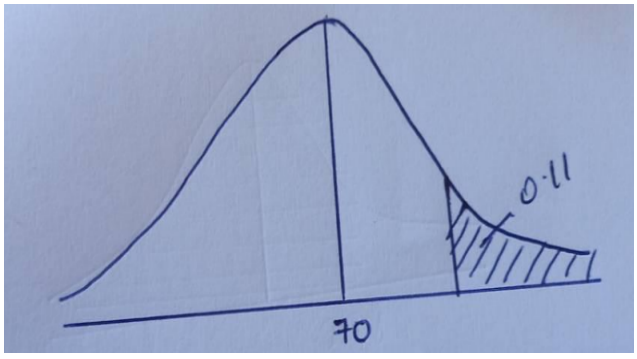
2. **Find** $P(z<k)=0.15$



0.5 0.15 = 0.35

P	z	P	z	P	z	P	z	P
.00	.0000	.10	.2533	.20	.5244	.30	.8416	.40
.01	.0251	.11	.2793	.21	.5534	.31	.8779	.41
.02	.0502	.12	.3055	.22	.5828	.32	.9154	.42
.03	.0753	.13	.3319	.23	.6128	.33	.9542	.43
.04	.1004	.14	.3585	.24	.6433	.34	.9945	.44
.05	.1257	.15	.3853	.25	.6745	.35	1.0364	.45
.06	.1510	.16	.4125	.26	.7063	.36	1.0803	.46
.07	.1764	.17	.4399	.27	.7388	.37	1.1264	

3. **X has a Normal distribution with mean** $=70$**. Given** $P(x>80)=0.11$**. Find the standard deviation.**



P	z
.30	.8416
.31	.8779
.32	.9154
.33	.9542
.34	.9945
.35	1.0364
.36	1.0803
.37	1.1264
.38	1.1750
.39	1.2265

$$z=\frac{x}{\quad}$$

$$1.2265=\frac{80\quad 70}{\quad}$$

$$=\frac{10}{1.2265}$$

$0.5 \quad 0.11 = 0.39$

Exercise:

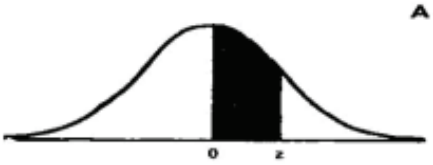
1. A normal random variable X has a mean of 50. Given that $P(x < 80) = 0.75$, find the standard deviation

2. A normal random variable has standard deviation of 50. Given that $P(x < 50) = 0.01$, find the mean

3. A normal distribution, X, has a standard deviation of 4. Given that $P(x < 70) = 0.7$, calculate the mean?

Strand	7 **PROBABILITY AND INFERENTIAL STATISTICS**
Sub Strand	7.2.1 Normal Distribution
Content Learning Outcome	Students should be able to; - Find probabilities using area under the normal probability curve.

Note: Not all probabilities are found under the inverse normal table, we have to use the normal distribution table.



AREAS UNDER NORMAL PROBABILITY CURVE

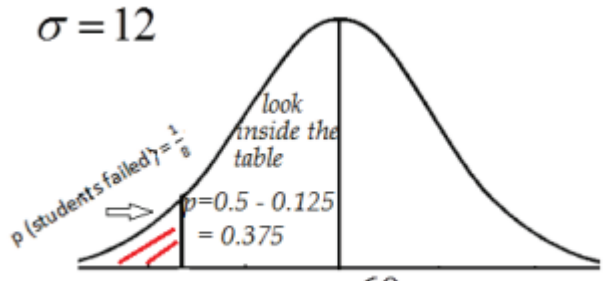
The tabulated value is the probability that the standardized normal variate Z (with μ=0, σ=1) lies between 0 and z.
e.g. P(0 < Z < 1.43) = 42.36%

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4	8	12	16	20	24	28	32	36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4	8	12	15	19	22	27	31	35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4	8	11	15	19	22	26	30	34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4	7	11	14	18	22	25	29	32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	3	4	5	6	7	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	4	5	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	1	2	2	2	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	0	1	1	1	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	0	1	1	1	1	1	2	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	0	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	0	0	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	0	0	0	0	0	0	0	0	0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0	0	0	0	0	0	0	0	0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995	0	0	0	0	0	0	0	0	0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

Example:

1. Fifteen thousand students sat for an exam and their marks were normally distributed with a mean of 64% and standard deviation of 12. One in eight students failed, what was the minimum mark required for a pass?

$P(\text{students failed}) = \frac{1}{8}$ or 0.125



$z = 1.151$

$z = \frac{x}{\quad}$

$1.151 = \frac{x \quad 64}{12}$

$x = 50.19$

z	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4 8 12	16 20 24	28 32 36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4 8 12	16 20 24	28 32 36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4 8 12	15 19 22	27 31 35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4 8 11	15 19 22	26 30 34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4 7 11	14 18 22	25 29 32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3 7 10	14 17 21	24 27 31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3 6 10	13 16 19	23 26 29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3 6 9	12 15 18	21 24 27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3 6 8	11 14 17	19 22 25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3 5 8	10 13 15	18 20 23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2 5 7	9 12 14	16 18 21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2 4 6	8 10 12	14 16 19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2 4 5	7 9 11	13 15 16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2 3 5	6 8 10	11 13 14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1 3 4	6 7 8	10 11 13

Exercise:

1. The final exam marks of a class of 500 students are normally distributed with a mean of 62 marks and standard deviation of 15 marks.
   a. What is the cut off mark for A- grade if the top 16.6% of the students in the class are awarded an A grade.
   b. The E – grade is the lowest in the examination. What is the cut – off mark for E -Grade if 2.55% students in the class are awarded an E- grade?

2. The marks for college examination are normally distributed with a mean of 56. If 6% of all students who set for the examination had marks **greater than 80,** find the standard deviation for the distribution.

Strand	7 **PROBABILITY AND INFERENTIAL STATISTICS**
Sub Strand	7. 3. 1 Binomial Probabilities
Content Learning Outcome	Students should be able to; - Define binomial experiments - Calculate binomial probabilities using formulae and tables.

Binomial Probabilities

The binomial distribution has **two possible outcomes** (the prefix "bi" means two).

Example: A coin has only two possible outcomes: heads or tails and taking a test has two possible outcomes: pass or fail.
The two outcomes are called **success or failure.**

- Properties of Binomial Experiment:
  1. The experiment consists of 'n' repeated trials
  2. Only two possible outcomes (Success or Failure)
  3. Probability of success is the same for each trial.
  4. Each trial is independent of each other

- The binomial distribution formula is:

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \quad , x = 0, 1, 2, 3, \ldots\ldots\ldots n$$

where:

P = binomial probability
x = total number of "successes"
p = probability of success
q = probability of failure ($q = 1 - p$)
n = number of trials

REFER TO PAGES 165 AND 166 FOR THE TABLE OF BINOMIAL

EXAMPLES

1. Penang soccer team has a probability of 0.4 of winning each game it plays. The team plays a total of 5 games. Find the probability that the team wins

i) Exactly two games

$n = 5$

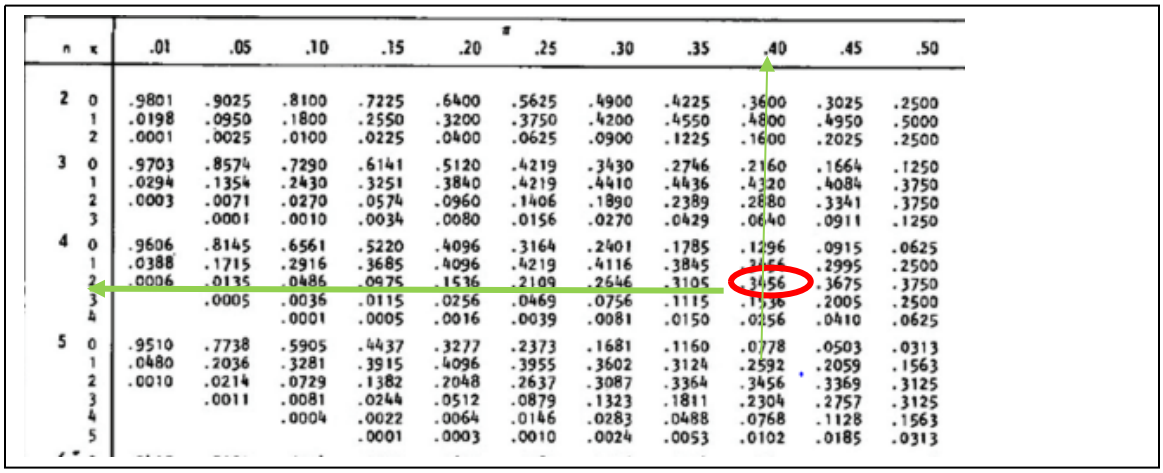
$p = 0.4$

$q = 1 \quad 0.4 = 0.6$

$(0.4)^2 0.6^3$

$= 10 \quad 0.16 \quad 0.216$

$= 0.3456$



n	x	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
2	0	.9801	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500
	1	.0198	.0950	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000
	2	.0001	.0025	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
3	0	.9703	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250
	1	.0294	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750
	2	.0003	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750
	3		.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250
4	0	.9606	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625
	1	.0388	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500
	2	.0006	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750
	3		.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500
	4			.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625
5	0	.9510	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313
	1	.0480	.2036	.3281	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563
	2	.0010	.0214	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125
	3		.0011	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125
	4			.0004	.0022	.0064	.0146	.0283	.0488	.0768	.1128	.1563
	5				.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313

ii) At least four games.

$P(X=4)=0.0768$

$P(X=5)=0.0102$

$0.0768+0.0102=0.0870$

2. It is found that 75% of the patients are suffering from a particular disease are cured successfully. What is the probability that 5 of the next 4 patients will be cured successfully?

$n=4$

$x=3$

$p=0.75$

$(0.25)^1(0.75)^3$

$=0.4219$

2. A hospital with a heart transplant unit finds that the probability that a patient is till alive after 3 years is 30%. Find the probability that if the unit operates on 10 patients in a year, more than 7 patients will not live up to 3 years after the operation.

3. A shop owner has found out that 80% of the people who came to his shop to buy something. If one day 20 people go to the shop, find the probability that at least 75% of them will be buying something.

THE END