PENANG SANGAM HIGH SCHOOL

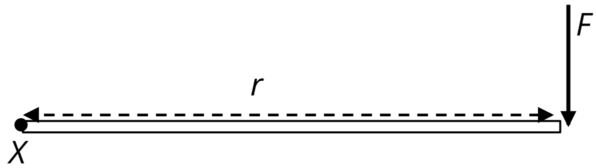
WEEK 16 **LESSON NOTES**

PHYSICS – Y13

STRAND: MECHANICS
SUB-STRAND: ROTATIONAL DYNAMICS
CONTENT LEARNING OUTCOME: To identify rotational motion and solve problems.

Torque (τ)

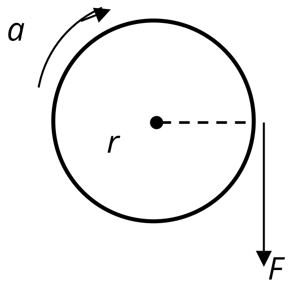
Torque (moments) is the turning effect of a force. A force applied at a distance from the pivot produces torque, causing rotation.



The torque about the pivot X is the product of the force F and the moment arm (perpendicular distance) r.

$$\tau = Fr \qquad \text{unit: Nm}$$

The diagram below shows a string wrapped around a wheel of radius r that is free to rotate about its axle. A force F is applied at the end of the string.



The force F produces a torque about the axle, given by:

$$\tau = Fr$$

As a result of the applied torque, the wheel starts to rotate, undergoing angular acceleration.
This means that the applied torque causes angular acceleration.

The applied torque, τ, is directly proportional to the angular acceleration, α.

$$\tau \propto \alpha$$

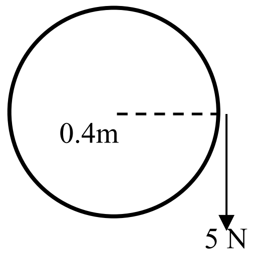
The constant of proportionality is the **moment of inertia**, I.

$$\tau = I\alpha \qquad \text{[Newton's 2nd law for rotation]}$$

τ = torque (Nm) I = inertia (kgm^2) α = angular acceleration (rad/s^2)

Eg

A 5 N force is applied to a wheel of radius 0.4 m and inertia 0.75 kgm^2.



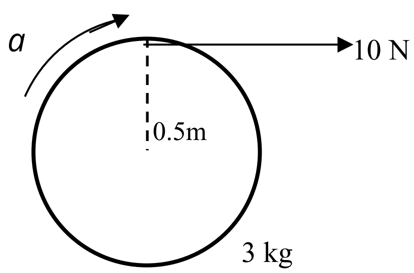
a) Calculate the torque on the wheel.

b) Calculate the wheel's angular acceleration.

c) If the wheel starts from rest, what angle does it turn through in 10 seconds?

Eg 2

A string, wrapped around a solid cylinder of mass 3 kg and radius 0.5 m, is pulled with a constant force of 10 N. The cylinder is free to rotate about its axis.



a) Calculate the torque produced by the applied force.

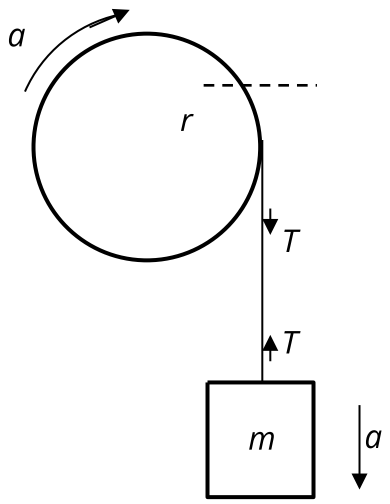
b) Calculate the inertia of the cylinder.

c) Calculate the angular acceleration of the cylinder.

d) If the cylinder starts from rest, what angle does it turn through in 4 seconds?

Rotation caused by falling mass

The diagram shows a cord wrapped around a wheel that is free to rotate about its axis. The other end of the cord is tied to a mass m, which is then released.



The hanging mass, m, will cause the wheel to rotate. The falling mass undergoes **linear motion** whereas the wheel undergoes **rotational motion**.

Since the falling mass, m, undergoes linear motion, we write the **linear form of Newton's 2nd law for it**.

ie. $F_{net} = ma$

The wheel undergoes rotational motion, therefore we write the **rotational form of Newton's 2nd law for it**.

ie. $\tau = I\alpha$

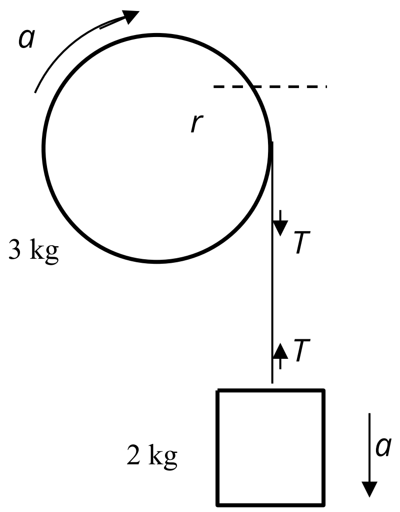
Note

The torque on the wheel is provided by the tension in the string.

ie. $\tau = Tr$

Eg 3

A cord is wrapped several times around a pulley of mass 5 kg that is free to rotate about its axis. The other end of the cord is tied to a 2 kg mass and released.



For the pulley, $I = \frac{1}{2}mr^{2}$.

a) Write an equation for the linear motion of the 2 kg mass.

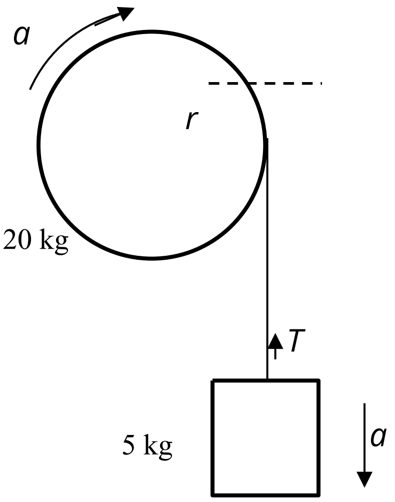
b) Write an equation for the rotational motion of the pulley.

c) Using the two equations above, eliminate T and find the linear acceleration of the system, a.

d) Calculate the tension in the cord.

Eg 4

A cord, tied to a 5kg mass, is wrapped several times around a solid sphere of mass 20 kg that is free to rotate about its axis. The mass is then released from rest.



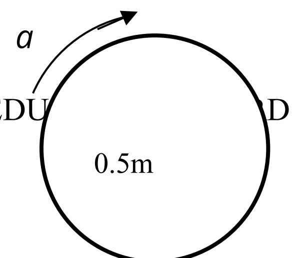
For the sphere, $I = \frac{2}{5}mr^2$.

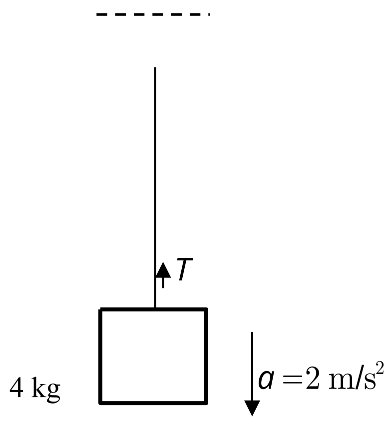
a) Calculate the linear acceleration of the system

d) Calculate the tension in the cord.

Eg 5

A cylindrical pulley of radius 0.5 m has a light string wound around it. A 4 kg mass hangs from the other end of the string as shown.





Calculate:

a) The tension in the string.

b) The torque acting on the pulley.

c) The angular acceleration of the pulley.

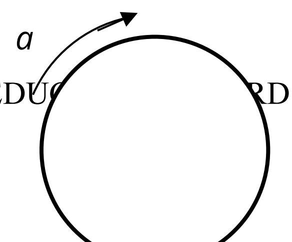
d) The rotational inertia of the pulley.

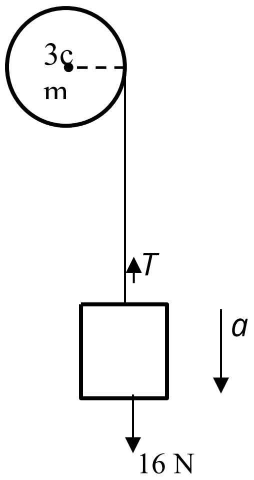
e) The mass of the pulley. [For the pulley, $I = \frac{1}{2}mr^2$]

<u>Eg 6</u>

A wheel and axle is caused to rotate about a horizontal axis by means of a 16 N weight attached to a cord wrapped around the axle of radius 3 cm.

The weight falls vertically through 7.2 m in six seconds starting from rest.

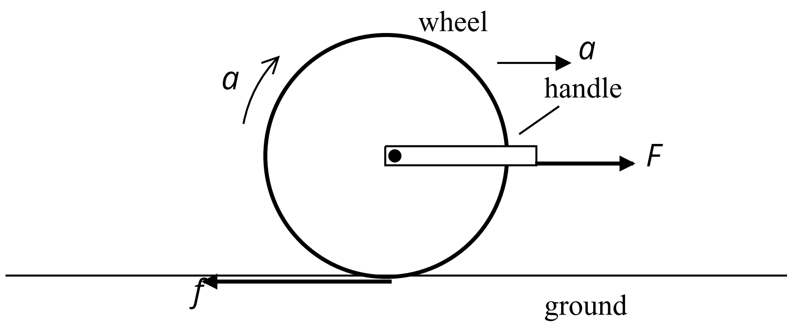




Determine the moment of inertia of the wheel and axle.

<u>Object undergoing both Linear and Rotational motion</u>

The diagram shows a wheel of mass m pulled by a constant force F along the ground by means of a light handle.



The wheel pulled along the ground with a constant force F undergoes **both linear and rotational motion**.

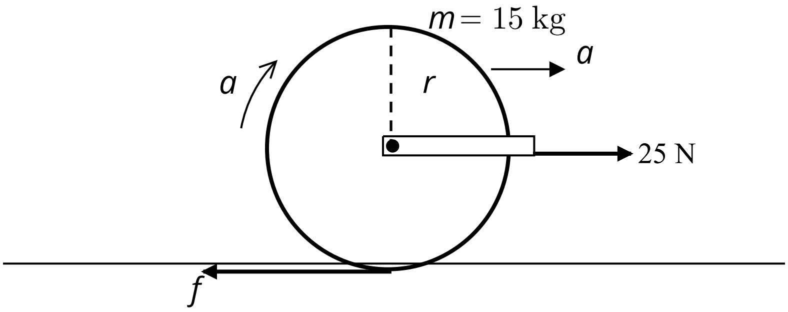
A frictional force f acts between the wheel and the ground which prevents the wheel from sliding.

Since the wheel undergoes both linear and rotational motion, we write both forms of Newton's 2nd law for the wheel.

Linear motion $F_{net} = ma$ Rotational motion $\tau = I\alpha$

<u>Eg 1</u>

A solid cylindrical drum of mass 15 kg is pulled over a rough horizontal surface by means of a light handle attached to a smooth axle as shown.



The pull on the handle is 25 N horizontally and the force of friction, f, which prevents the drum from sliding has the direction shown.

For the drum, $I = \frac{1}{2}mr^2$.

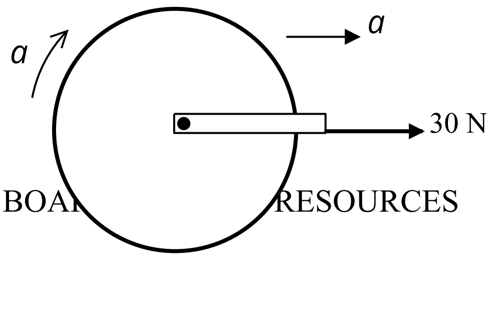
Calculate:

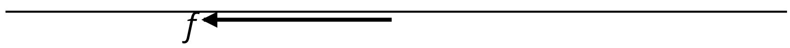
a) The linear acceleration of the drum, a.

b) The magnitude of the frictional force, f.

Eg 2

A solid sphere $\left(I = \frac{2}{5}mr^2\right)$ of mass 10 kg is pulled with a constant horizontal force of 30 N as shown.

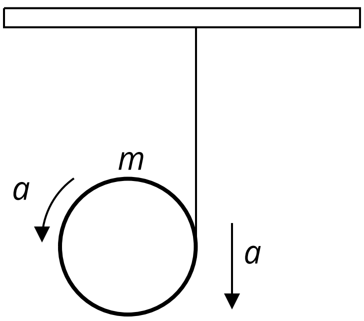




a) Calculate the linear acceleration of the sphere.

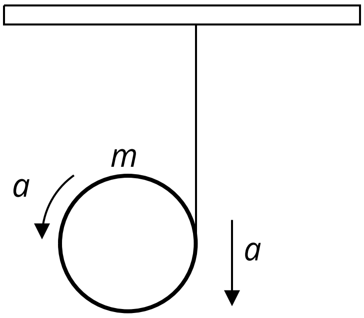
b) Calculate the frictional force, f.

<u>Eg 3</u>
A uniform disc has a mass m and radius r. A smooth light string is securely fixed to it and wrapped around its circumference. The free end is tied to the ceiling as shown.



The disc is released from rest. For the disc, $I = \frac{1}{2}mr^2$.

a) On the diagram, draw vectors to show the direction of weight force, W, and the tension, T, acting on the disc.

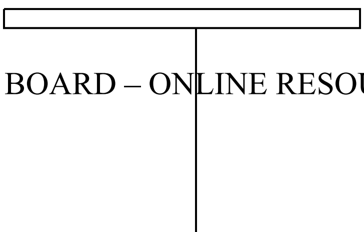


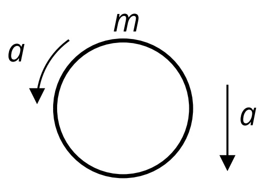
b) Write an equation for the linear motion of the disc in terms of m, g, a and T.

c) Write an equation for the rotational motion of the disc in terms of m, a, r and T.

d) Hence, by eliminating T from the equations, show that the linear acceleration of the disc is $a=\frac{2}{3}g$.

Eg 4





The sphere is released from rest. Show that the linear acceleration of the sphere is $a = \frac{5}{7}g$, where g is the gravitational constant.

For the sphere, $I = \frac{2}{5}mr^2$.