PENANG SANGAM HIGH SCHOOL

WEEK 17 **LESSON NOTES**

PHYSICS – Y13

STRAND: MECHANICS

SUB-STRAND: ROTATIONAL DYNAMICS

CONTENT LEARNING OUTCOME: To be able to identify a system having Rotational Kinetic Energy and using conservation of energy to solve problems

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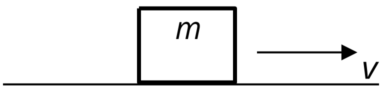
Rotational Kinetic Energy

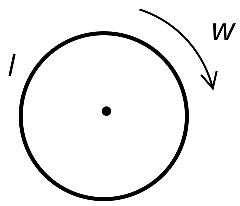
When an object undergoes linear motion, it has **translational** (linear) kinetic energy.



$$K_T = \frac{1}{2}mv^2$$

K_T = translational kinetic energy m = mass (kg) v = linear velocity (m/s

An object rotating about its axis has **rotational kinetic energy**.



The rotational kinetic energy of the object is given by:

$$K_R = \frac{1}{2}Iw^2$$

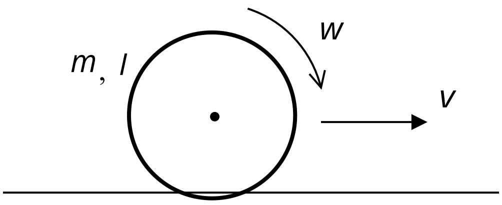
K_R = rotational kinetic energy (I = inertia (kgm^2) w = angular velocity (rad/

Eg 1
A wheel of inertia 15 kgm^2 rotates at a constant angular velocity of 4 rad/s. Calculate its kinetic energy.

Eg 2
Calculate the kinetic energy of a wheel of inertia 12 kgm^2 rotating at 3 rev/s.

Eg 3
Calculate the increase in kinetic energy when a disc of inertia 5 kgm^2 accelerates from 3 rad/s to 7 rad/s.

Note
An object undergoing **both** linear motion and rotational motion (eg drum rolling on ground) has both **translational** and **rotational kinetic energies**. The total kinetic energy of the object will be the sum of the two types of kinetic energies.

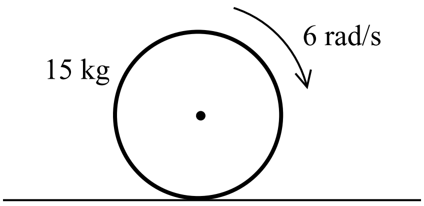


ie. $E_{K\,TOTAL} = K_T + K_R$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Eg 1

A solid cylinder $\left(I = \frac{1}{2}mr^2\right)$ of mass 15 kg and radius 0.5 m is rolling along the ground at 6 rad/s.



Calculate:

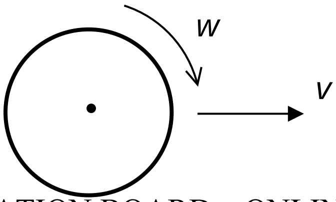
a) Its rotational kinetic energy.

b) Its translational kinetic energy.

c) Its total kinetic energy.

Eg 2
A solid sphere of mass, m, and radius, r, rolls along the ground with a linear speed v and an angular speed ω.

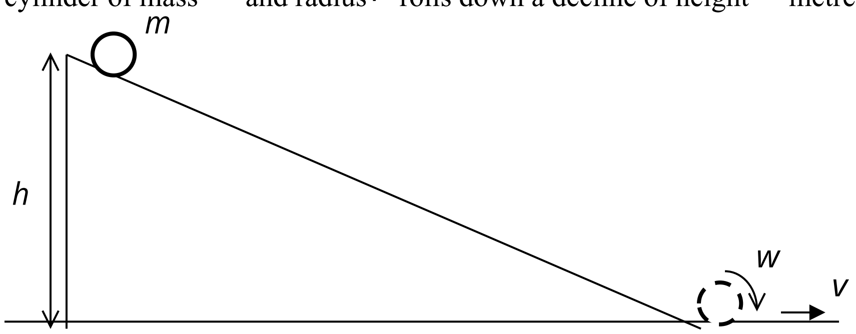
For the sphere, $I = \frac{2}{5}mr^2$.



Show that the total kinetic energy of the sphere is given by $E_K = \frac{7}{10}mv^2$, where m is its mass and v is its linear speed.

Eg 3

A solid cylinder of mass m and radius r rolls down a decline of height h metres.



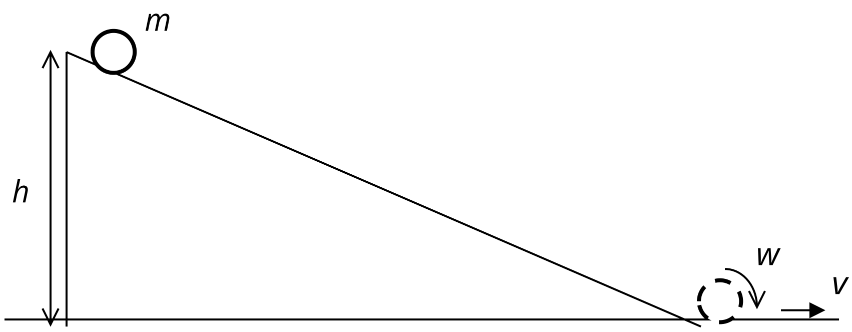
For the cylinder, $I = \frac{1}{2}mr^2$.

a) Show that the total kinetic energy of the cylinder at the bottom of the incline is $E_K = \frac{3}{4}mv^2$, where v is the linear speed at the foot of the incline.

b) Show that the linear velocity is given by $v = \sqrt{\frac{4gh}{3}}$.

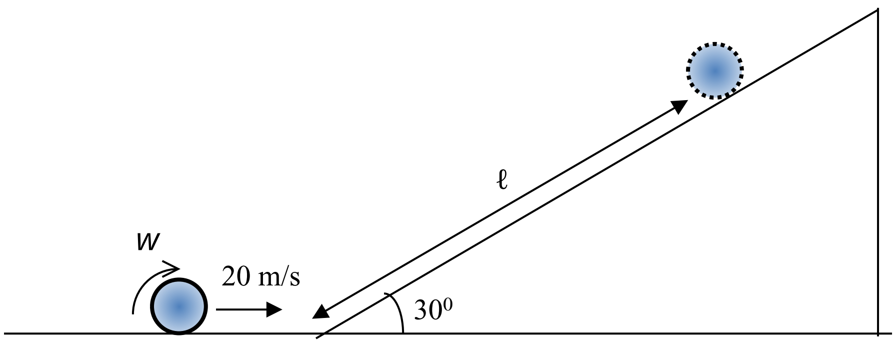
Eg 4

A steel ball of mass m and radius r starts from rest at a height, h metres and rolls down an inclined plane as shown.



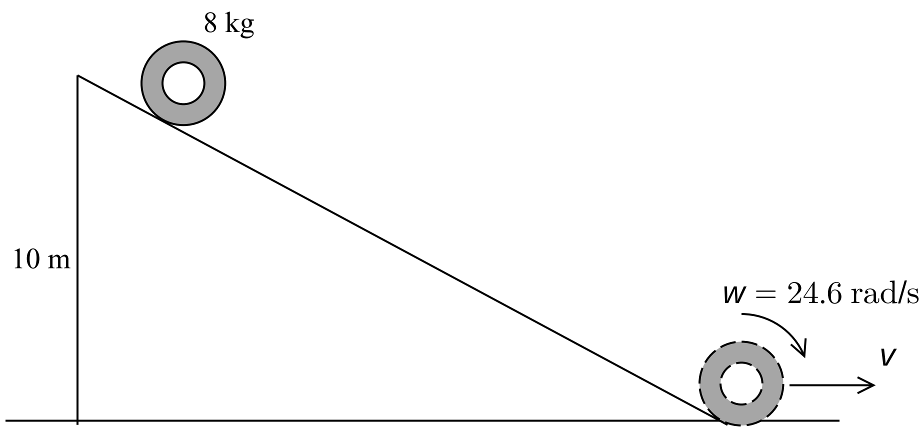
For the ball, $I = \frac{2}{5}mr^2$.
When the ball leaves the incline, the initial potential energy is converted to rotational and translational kinetic energy. Show that the linear velocity of the ball at the bottom of the incline is $v = \sqrt{\frac{10gh}{7}}$.

Eg 5
A 600 g solid ball rolling on a horizontal surface at 20 m/s comes to the bottom of an inclined plane which makes an angle of 30^0 to the horizontal.



For the ball, $I = \frac{2}{5}mr^2$.
Neglecting friction, how far up the incline will the ball roll?

Eg 6
An 8 kg car wheel rolls down a uniform slope 10 m high. The translational kinetic energy of the wheel when it reaches the bottom of the slope is 444 J. The angular speed of the wheel when it reaches the bottom of the slope was 24.6 rad/s.



Calculate:

a) The linear speed, v, of the wheel at the bottom of the incline.

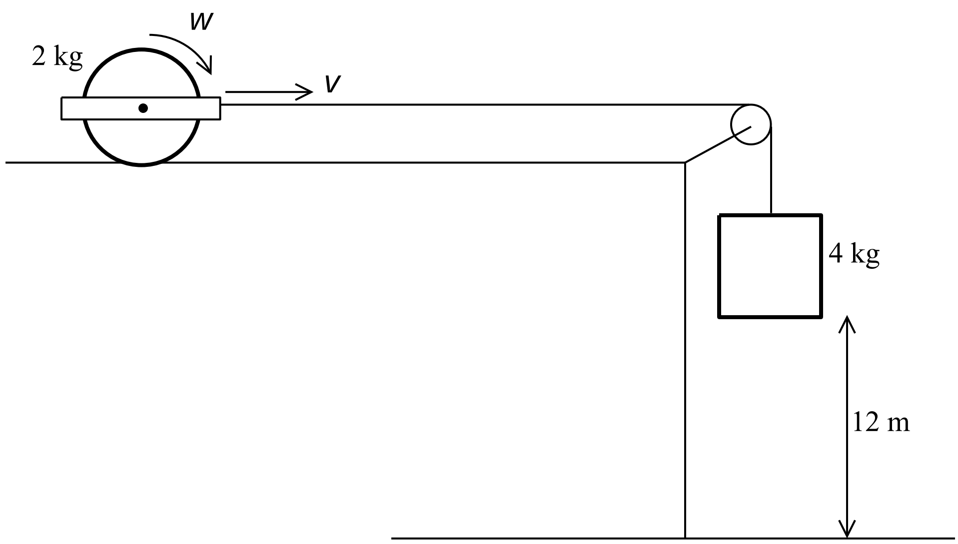
b) The radius of the wheel.

c) The rotational kinetic energy of the wheel at the bottom of the slope.

d) The rotational inertia of the wheel.

Eg 7

A 4 kg hanging mass is connected to a 2 kg cylinder bracket over a frictionless pulley as shown.



It is found that when the hanging mass falls 12 m, the system has a linear speed of 8 m/s.
Calculate:

a) The potential energy lost by the system.

b) The translational kinetic energy gained by the system,

c) The rotational kinetic energy gained by the cylinder bracket.

d) The angular velocity of the cylinder if its radius is 0.1 m.

e) The inertia of the bracket.

Work done in Rotational Motion

Linear motion: $W = Fs$.

W = w ork (J $\quad F$ = force (N) $\quad s$ = linear displacement (n

Rotational motion: $W = tq$

W = w ork (J $\quad t$ = torque (Nm $\quad q$ = angular displacement (ra

Eg 1
Calculate the work done on a wheel by a 10 Nm torque that rotates the wheel through 15 rad.

Eg 2
Calculate the work done when a 7 Nm torque turns a wheel through 5 revolutions.

Note
When work is done on an object, its kinetic energy changes. The change in kinetic energy is equal to the work done on the object.

ie. $W = DE_K$

Eg 3
Calculate the work done on a flywheel of inertia 2.5 kgm^2 if it accelerates from 3 rad/s to 7 rad/s.

Eg 4
Calculate the work done when a wheel of inertia 15 kgm^2 accelerates from 3 rev/s to 7 rev/s.