3055 BA SANGAM COLLEGE

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WORKSHEET 20

School: Ba Sangam College **Year / Level:** 12

Subject: Mathematics **Name of Student:** _______________

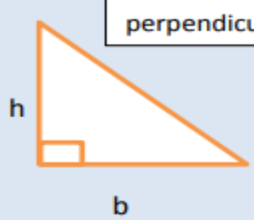
Strand	5 – Trigonometry
Sub strand	5.1 – Triangles
Content Learning Outcome	➢ Investigate and solve problems using trigonometric relations

Trigonometry **(Ref: Year 12 Mathematics Pg 147 -154)**

Area of a Triangle

Right - angled triangle

Given Base and perpendicular Height

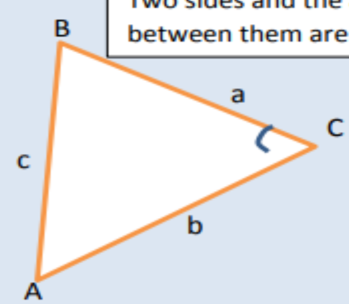


Area: A = ½ bh

Where b – base and
h – Perpendicular height

Non - right angled triangle

Two sides and the angle between them are known

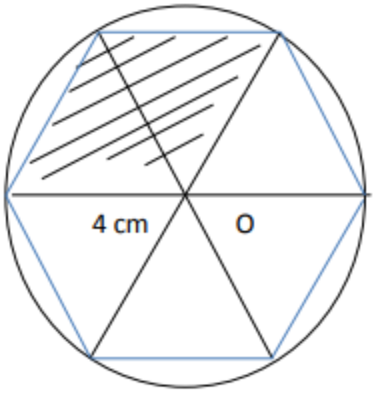


A = ½ ab Sin C or

A = ½ ac Sin B or A = ½ bc Sin A

Example 1:

The diagram below shows a regular hexagon inscribed in a circle of radius 4 cm at centre O. (Diagram not to scale).



Calculate:

a) Angle of each sector formed.
b) the area of one of the triangle
c) the area of shaded region.

a) Angle region

$$T_{angle} = 360^0$$
$$6x = 360^0$$
$$x = 60^0$$

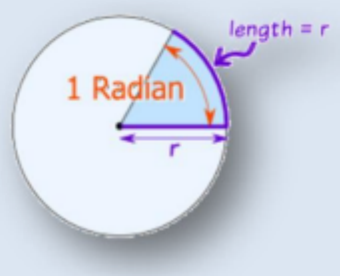
b) area of one of the triangle

$$A_\Delta = \frac{1}{2}abSinC$$
$$= \frac{1}{2}(4\times4)Sin60^0$$
$$= 4\sqrt{3}cm^2$$

c) area of shaded

$$A = 2\times A_\Delta$$
$$= 2\times 4\sqrt{3}$$
$$= 8\sqrt{3}cm^2$$

Conversion to degrees to radians and vice versa



One complete circle = 360° or 2π radians

- From above, π radians = 180°
  2π radians = 360°

- To convert degrees to radians: multiply by π/ 180

$$\frac{\theta^\circ}{180^\circ}\times\pi = \text{angle in radians}$$

- To convert radians to degrees: divide by π/ 180 or multiply by 180 / π

$$\text{Angle in radians}\times\frac{180^\circ}{\pi} = \theta^\circ$$

Conversion of some common angles

The table shows the conversion of some **common angles**.

Units	Values										
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
Degrees	30°	45°	60°	90°	120°	135°	180°	225°	270°	315°	360°

Example 1: Convert the following angles to radians

a) $45°$

$$\text{angle} = \frac{\theta°}{180°} \times \pi$$
$$= \frac{45°}{180°} \times \pi$$
$$= \frac{\pi}{4}$$

b) $15°$

$$\text{angle} = \frac{\theta°}{180°} \times \pi$$
$$= \frac{15°}{180°} \times \pi$$
$$= \frac{\pi}{12}$$

c) $25°$

$$\text{angle} = \frac{\theta°}{180°} \times \pi$$
$$= \frac{25°}{180°} \times \pi$$
$$= \frac{5\pi}{36}$$

Example 2: Convert the following angles to degrees

a) $\frac{\pi}{5}$

$$\text{angle} = \theta \times \frac{180°}{\pi}$$
$$= \frac{\pi}{5} \times \frac{180°}{\pi}$$
$$= 36°$$

b) $\frac{5\pi}{2}$

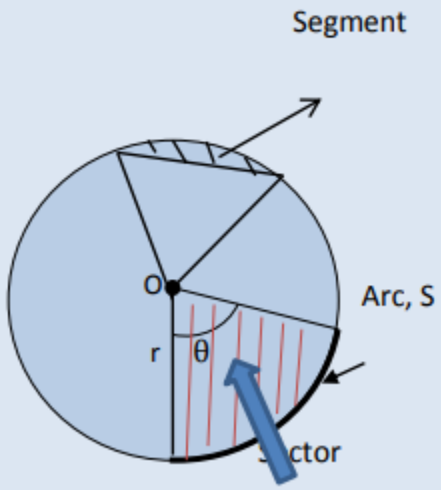
$$\text{angle} = \theta \times \frac{180°}{\pi}$$
$$= \frac{5\pi}{2} \times \frac{180°}{\pi}$$
$$= 450°$$

c) $\frac{4\pi}{7}$

$$\text{angle} = \theta \times \frac{180°}{\pi}$$
$$= \frac{4\pi}{7} \times \frac{180°}{\pi}$$
$$= \frac{720°}{7}$$
$$or\, 102.86°$$

ARC LENGTH, AREA OF SECTOR and SEGMENT

Note: Refer to the Parts of a circle



where r is the radius of the circle
θ is the angle in **radians**
O as **center** of circle

1. ARC LENGTH (S)

The arc length is the measure of the distance along the curved line making up the arc.

Formulae:

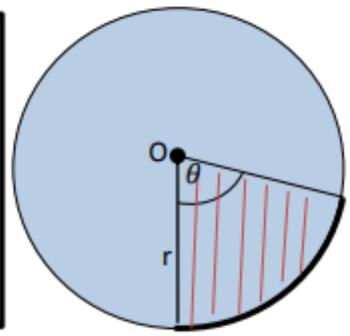
$S = r\theta$
[Angle must be in radians]

2. **AREA OF SECTOR**

Sector is the area enclosed by an arc and the two radii

Formulae:

Area $_{\text{sector}} = \frac{\theta}{360^\circ} \times \pi r^2$
Since 360° = 2π radians , substituting yields
Area $_{\text{sector}} = \frac{\theta}{2\pi} \times \pi r^2$
$= \frac{\theta}{2} \times r^2$, rearranging
$= ½ r^2\theta$
[angle must be in radians]

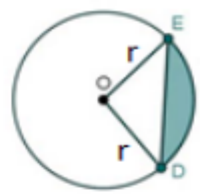


3. AREA OF SEGMENT

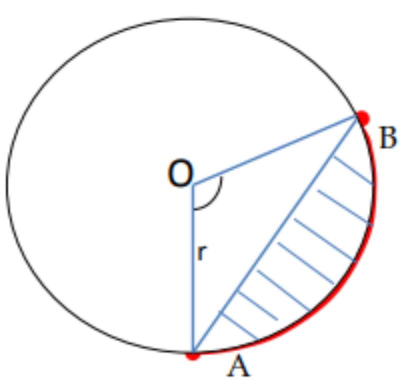
Segment is the region bounded by a chord and an arc.

Formulae:

Area $_{\text{segment}}$ = Area $_{\text{sector}}$ – Area $_{\text{triangle}}$
$= ½ r^2\theta - ½ r^2 \sin\theta$
or
$= ½ r^2(\theta - \sin\theta)$
[angle must be in radians]



Example 3: The diagram below shows a circle of radius r. OAB is a sector of the circle and has an angle of 95°.



i. Convert the angle to radians.

ii. Given that the segment has an area of 100 cm^2, calculate radius of the circle.

iii. Calculate the length of minor arc AB.

iv. Hence or otherwise, determine the perimeter of the sector.

Answers:

i. Conversion to radians

angle = $\frac{\theta^\circ}{180^\circ} \times \pi$

$= \frac{95^\circ}{180^\circ} \times \pi$

$= \frac{19\pi}{36}$

ii. radius of the circle

Area $_{segment}$ = ½ r^2(θ − sin θ)

100 = ½ r^2 ($\frac{19\pi}{36}$ − sin $\frac{19\pi}{36}$)

100 = 0.330934045 r^2

$\frac{100}{0.330934045} = r^2$

r^2 = 302.175

r = 17.38 cm

iii. Length of minor arc AB

add all sides

S = r θ

= 17.83 × $\frac{19\pi}{36}$

= 28.82 cm

iv. Perimeter of the sector:

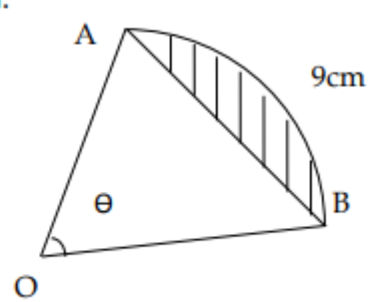
P = $\overline{OA} + \overline{OB} +$ Length of arc AB

= 17.38 + 17.38 + 28.82

= 63.58 cm

ACTIVITY

1. The diagram below shows a sector of a circle with the radius of 6cm and the length of the arc is 9cm.



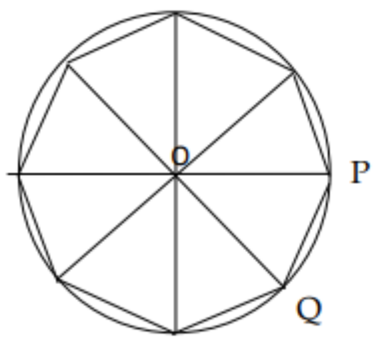
a. Show that the angle ϴ = 1.5 rad.

b. Calculate the area of the sector OAB.

c. Calculate the area of the shaded segment.

(6 MARKS)

2. The diagram below shows a regular octagon inscribed in a circle of radius 4 cm with centre O.



Calculate the following:

a. Length of arc PQ
b. Area of sector OPQ
c. Area of triangle OPQ
d. Area of the octagon

(2 MARKS)

THE END