PENANG SANGAM HIGH SCHOOL

WEEK 21 LESSON NOTES

PHYSICS – Y13

STRAND 5: DIRECT CURRENT

SUB-STRAND: **Kirchoff's Laws**

CONTENT LEARNING OUTCOME: To apply Kirchoff's Laws and determine the current flow and voltages in an electrical circuit.

Kirchoff's Laws

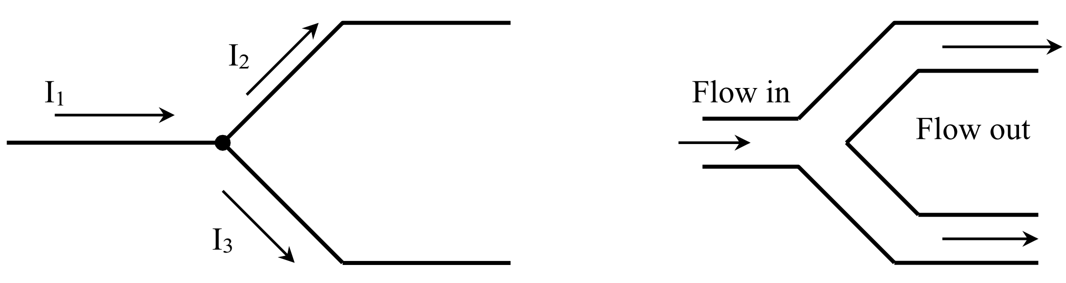
Simple circuits are analysed using Ohm's law. More complex circuits containing several sources of ***emf*** and resistances are analysed using Kirchoff's laws.

Kirchoff's Rule 1. (The junction rule)

"The sum of currents entering any junction must be equal to the sum of the currents leaving that junction."

This junction rule is based on the ***conservation of charge***.

The schematic diagram below illustrates Kirchoff's junction rule. This rule requires that whatever current enters a junction must leave that junction. This is analogous to liquid flowing through a pipe and splitting at a junction. The total amount of liquid entering the junction in the pipe should equal the liquid carried by the branching pipes.

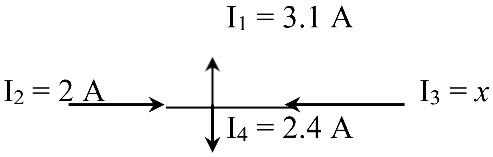


(a) Schematic diagram
(b) A mechanical analogue

If we apply the junction rule to the above schematic diagram, we get $I_1 = I_2 + I_3$.

Example.

Calculate the unknown current in the circuit given.



Using the junction rule, we get:

$I_2 + I_3 = I_1 + I_4$

$x = I_1 + I_4 - I_2$

$x = (3.1 + 2.4 - 2)$ A

$\underline{x = 3.5 \text{ A}}$

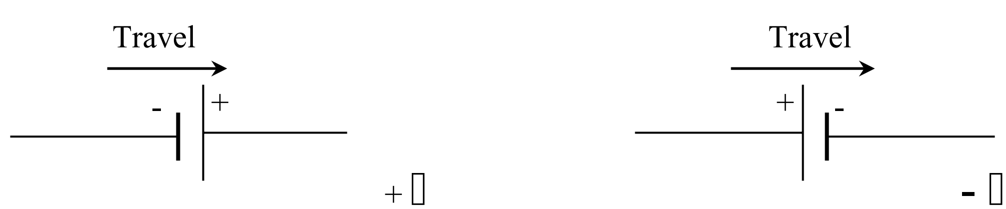
Kirchoff's rule 2. (The loop rule)

"The sum of the potential differences across all elements around any closed-circuit loop must be zero."

The loop rule is based on the principle of ***conservation of energy***.
Any charge moving around any closed loop in a circuit must gain as much energy as it loses.

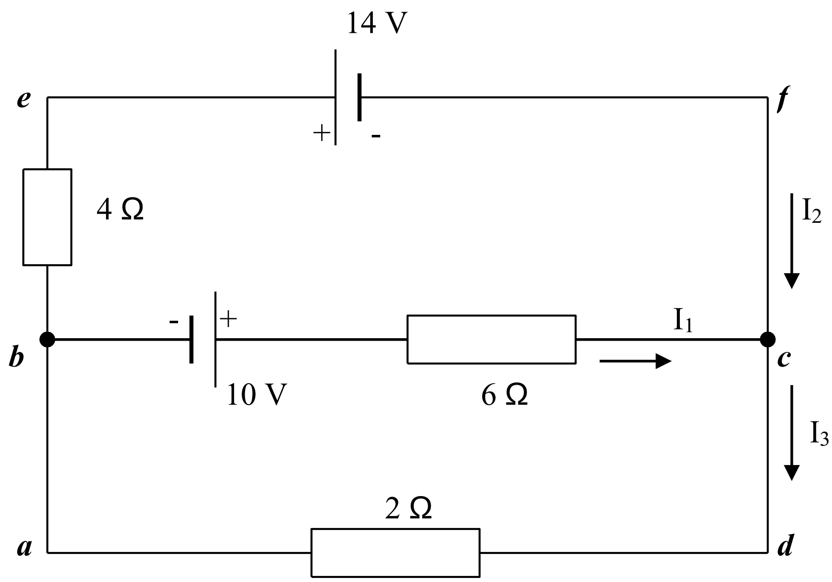
Problem solving strategy.

1. Assign symbols and directions to the currents in all the branches of the circuit. If the direction of the current is labelled wrongly then after analysing, the value will be negative but the magnitude of the current will be accurate.
2. Choose a junction and use the first rule to relate all the currents.
3. Choose any closed loop in the network and designate a direction (clockwise or counter clockwise). It doesn't have to be in the direction of the assumed current.
4. Start from a point and traverse the loop and write the elements as emf's using the following the rules:
   a. An emf is positive if you traverse it from – to + and negative if traversed from + to –.
   b. An *IR* product is negative if your path is in same direction as the assumed current and positive if direction of current is opposite to chosen path.
5. Solve the equations simultaneously for the unknown quantities. Be careful in your algebraic steps, and check your numerical answers for consistency.



Example

Find the currents I_1, I_2 and I_3 in the multi-loop circuit given.



Solution.

To find the three unknown currents, we apply the junction rule at one junction and the loop rule twice to obtain three equations and so be able to solve for three variables.

Step i. Choosing junction ***c*** and applying the Kirchoff's first rule we get

$$I_1 + I_2 = I_3 \qquad (1)$$

Step ii. The circuit has three loops but only two is needed, so lets take loops ***abcda*** and ***befcb*** and traverse in a clockwise direction

Loop abcda: $10\text{ V} - 6\,I_1 - 2\,I_3 = 0$ (2)

Loop befcb: $-14\text{ V} + 6\,I_1 - 10\text{ V} - 4\,I_2 = 0$

Loop *befcb* simplifies to $-24\text{ V} + 6\,I_1 - 4\,I_2 = 0$ (3)

Step iii. We now have to use the equations 1, 2 and 3 and solve them simultaneously. Take equation 1 and substitute in equation 2.
(The units can be ignored in the calculations)

$$10 - 6\,I_1 - 2\,(I_1 + I_2) = 0$$

$$10 - 8\,I_1 - 2\,I_2 = 0 \qquad (4)$$

Step iv. We have to use the equations (3) and (4) and eliminate one of the variables. Take equation (3) and divide throughout by 2.

$$12 - 3\,I_1 + 2\,I_2 = 0 \qquad (5)$$

Step v. Add equation (5) to (4) to eliminate I_2, gives

$$12 - 3\,I_1 + 2\,I_2 = 0$$

$$(+) \qquad 10 - 8\,I_1 - 2\,I_2 = 0$$

$$22 - (11)\,I_1 = 0 \quad , \quad (11)\,I_1 = 22 \; , \; \underline{I_1 = 2\text{ A}}$$

Step vi. Substituting I_1 in (5) results in a value for I_2

$$12 - 3\,(2) + 2\,I_2 = 0 \quad , \quad I_2 = -3\text{ A}$$

Step vii. Finally use equation (1) to calculate I_3

$$I_3 = I_1 + I_2 \quad , \quad I_3 = -1\text{ A}$$

The values for the currents are:

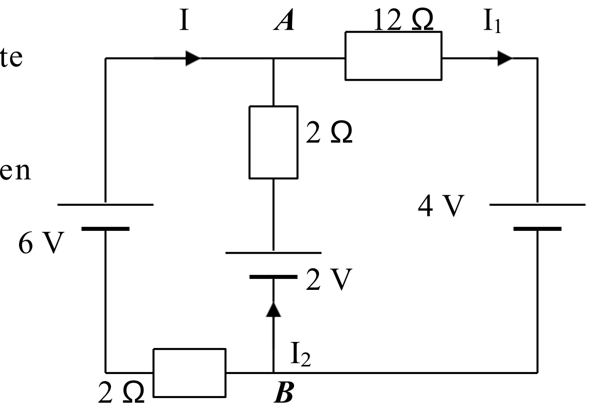
$I_1 = 2$ A $\qquad I_2 = -3$ A $\qquad I_3 = -1$ A

The **negative** values of I_2 and I_3 indicate that the directions of the currents are **opposite** to that designated initially.

1. Use the circuit below to answer the questions that follow.

   (i) Use the Kirchoff's Laws to calculate the values of I, I_1 and I_2.

   (ii) Find the potential difference between Points A and B.



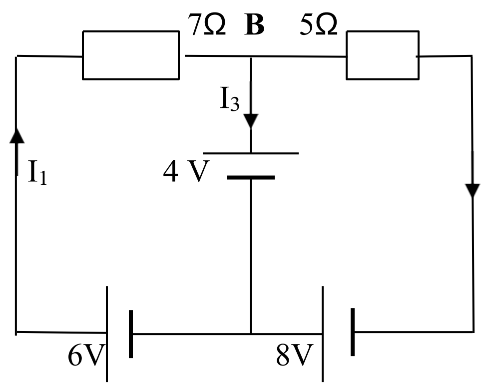
i)
ii)

2. A multiloop circuit is given below.

(i) Using junction ***B***, write down the Equation relating the currents.

(ii) Calculate values of I_1 , I_2 and I_3 .

Comment on significance of any negative values obtained.



i)
ii)

Electrical conductivity is understood on the basis of the mobility or lack of mobility of electrons in a material.

Metallic crystals

The valence electrons are not bound to individual lattice sides but are free to move through the crystal. Thus metals are good conductors.

Example: Mg – $1s^2\ 2s^2\ 2p^6\ 3s^2$ electrons in $3s^2$ are free to move

Covalent crystals

The valence electrons are involved in bonds responsible for the crystal structure and are therefore not free to move. Thus there are no mobile charges for conduction and such materials are insulators. *Example*: CO_2 , $O = C = O$

Ionic crystals

The charges are held in fixed positions when in the crystalline solid state and so is an insulator. However in the molten state the ions (charges) are mobile and so is a conductor.

Example: NaCl (common salt)

Temperature and Conductors

	Temperature	**Resistivity**	**Conductivity**
Insulator	Increase	Decrease	Increase
Conductor	Increase	Increase	Decrease
Semi-conductor	Increase	Decrease	Increase

The resistivity (ρ) of a good insulator is much greater than that of a good conductor by an enormous factor. Example: ρ *(copper)* = $1.7 \times 10^{-8}\ \Omega m$

ρ *(Pyrex)* = $10^{12}\ \Omega m$

Metals

Lattice vibration increases with raise in temperature and provides a larger surface area for the electrons to collide. The increased rate of collision of electrons increases the resistivity and therefore decreasing the conductivity (conduction).

Insulators

In insulators, what little conduction does take place is due to electrons that have gained enough energy from thermal motion of the lattice to break away from their “home” atoms and wander through the lattice.

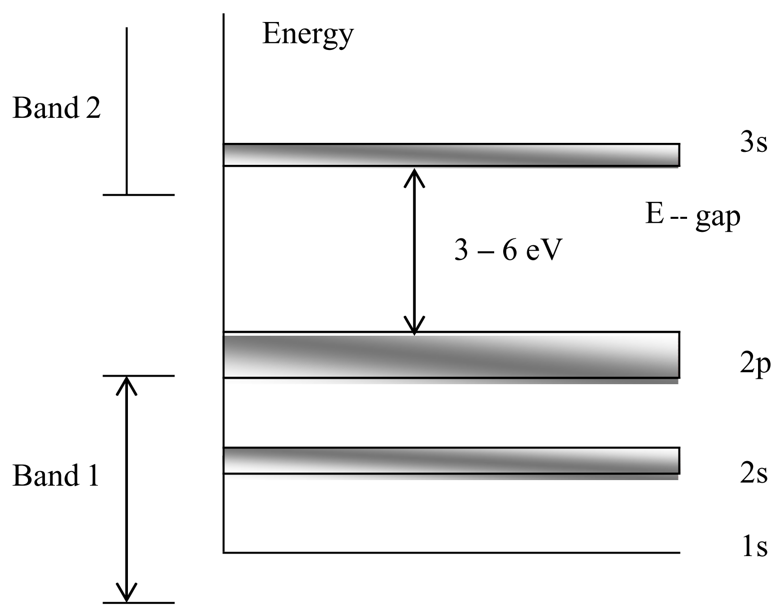
Semi – conductors

This is a class of materials intermediate between conductors and insulators in their ability to conduct electricity. (Example: Silicon (Si) and Germanium (Ge)) – group 4 elements in the periodic table. All the electrons are involved in bonding but a relatively small amount of energy is needed to break electrons from the lattice and to become mobile. So as the temperature increases, the conductivity of a conductor increases.

Band Structures

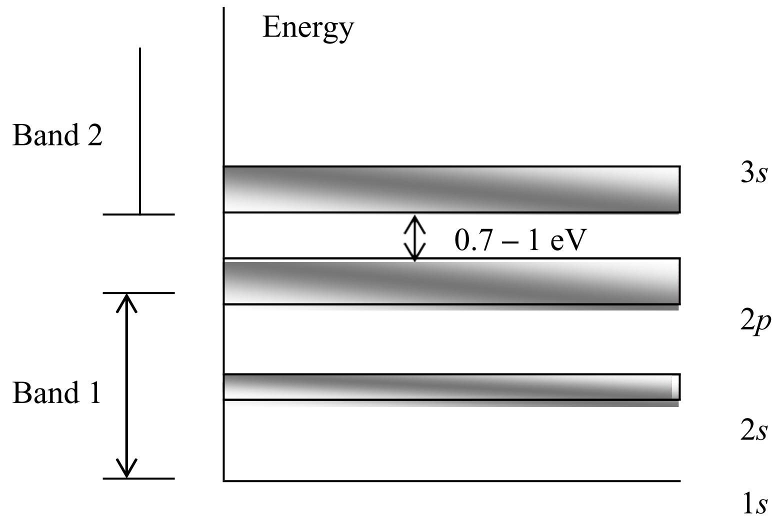
1. *Insulators*

For insulators, band 1 is completely filled but band 2 is too far above band 1 energetically to permit any appreciable number of band 1 electrons to jump the energy gap. (E – gap).



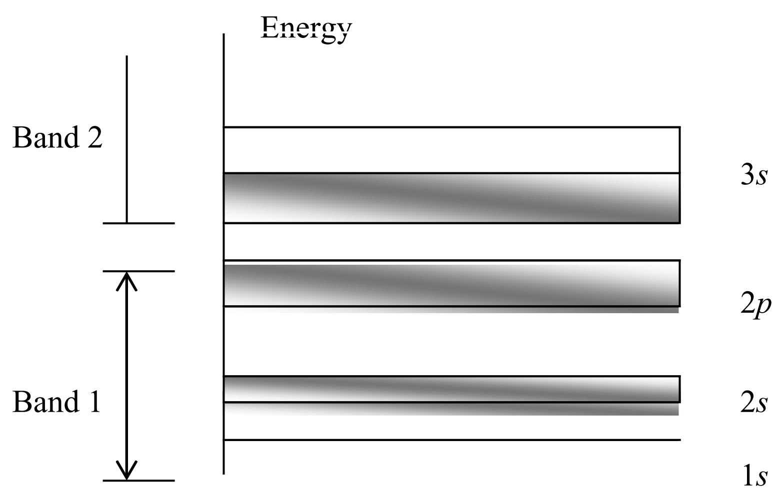
2. *Semi – conductors*

In semi – conductors such as silicon, band 1 is completely filled but band 2 is so close energetically that electrons can jump easily after absorbing energy into the unfilled levels of that band.



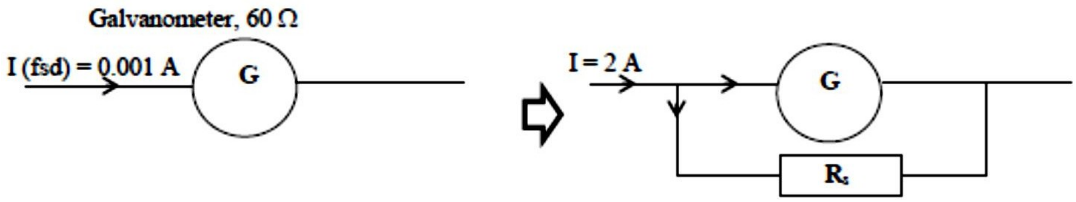
3. *Conductors*

Conduction in conductors such as copper have their band 2 only partially filled so that the electron can move easily to the higher energy levels and thus travel through the solid.



FSFE 2015
SHORT ANSWER (QUESTION 3)

A student wants to convert a galvanometer of internal resistance 60 Ω and full scale deflection (fsd) of 0.001 A to an ammeter that can read a maximum of 2 A. He connects a shunt resistance in parallel to the galvanometer to carry bulk of the current as shown below.



(i) Calculate the maximum voltage across the galvanometer. **(1 mark)**
(ii) Calculate the shunt resistance, **R_s**, needed for the conversion. **(2 marks)**

i)	ii)

LONG ANSWER(QUESTION 2)

(b) A 0.2 m long silver wire transfers 80 C of charges in 60 minutes.

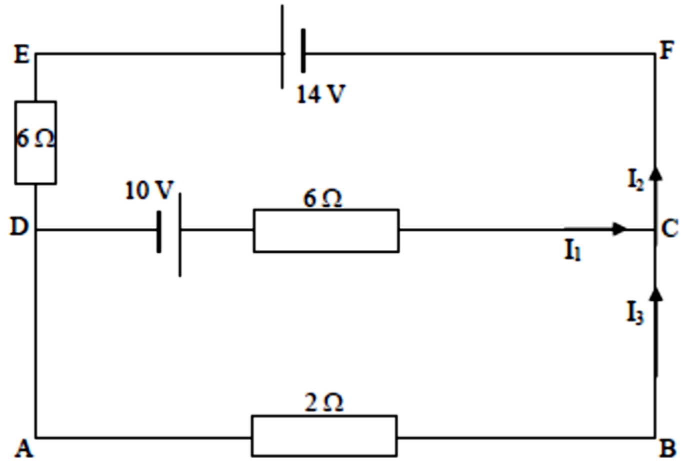
(i) Calculate the current in the wire. **(2 marks)**

(ii) If the silver wire is cylindrical with a radius of 1 mm and silver contains 5.2 x 10^{28} free electrons per cubic meter, calculate the number of free electrons in the 0.2 m long wire. **(3 marks)**

i)
ii)

LONG ANSWER(QUESTION 6)

(a) A multi-loop circuit is given below.



(i) Using junction C write the equations relating the currents I_1, I_2 and I_3. **(1 mark)**

(ii) Using Kirchoff's Law, write the equation for the loop ABCD. **(2 marks)**

i) & ii)

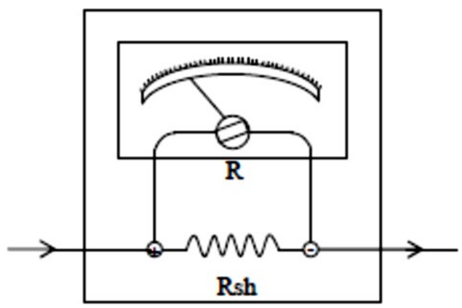
(b) A copper conductor 1 meter long has a uniform cross-sectional area of 3×10^{-6} m^2, and carries a current of 21 A. If the conductor has 10^{29} electrons per m^3, calculate the drift velocity of the electrons. **(2 marks)**

b)

FSFE 2014

SHORT ANSWER QUESTION 2

A current measuring instrument is usually called an ammeter. The diagram below shows the internal connections of a moving-coil ammeter. The parallel resistor is called a shunt resistor or simply a shunt, denoted as Rsh.



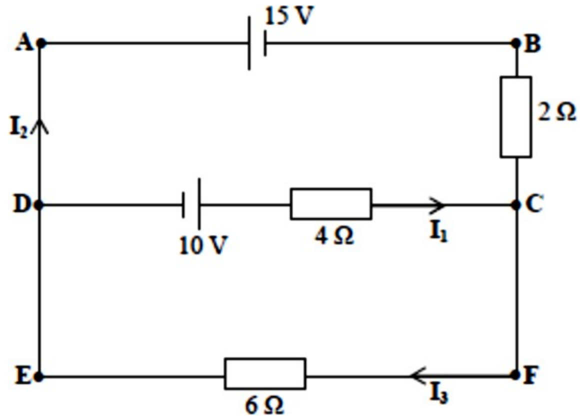
(i) Calculate the shunt resistance required to make the 1 mA, 15 Ω meter described above into an ammeter with a range of 0 A to 40 mA. **(2 marks)**

(ii) Calculate the total resistance (or equivalent resistance) for the ammeter above. **(1 mark)**

i) & ii)

LONG ANSWER QUESTION 5

(a) Use the multiloop circuit given below to answer the following questions.



(i) Write the current equation relating to the currents **I_1**, **I_2** and **I_3** at junction **D**. **(1 mark)**

(ii) Using Kirchoff's Law, write the equation for loops:

I. ABCDA and II. **DEFCD** **(2 marks)**

(iii) Use the equations in parts (i) and (ii) to calculate the three unknown currents: **I_1**, **I_2** and **I_3**. **(2 marks)**

LONG ANSWER QUESTION 7

(a) **Diagram 1** below shows a source (a battery) with ε = 14 V and of r = 3 Ω. **Diagram 2** shows how the source in **Diagram 1** is added to a R = 5 Ω resistor to form the complete circuit.

Diagram 1

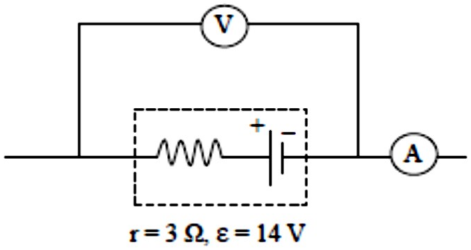
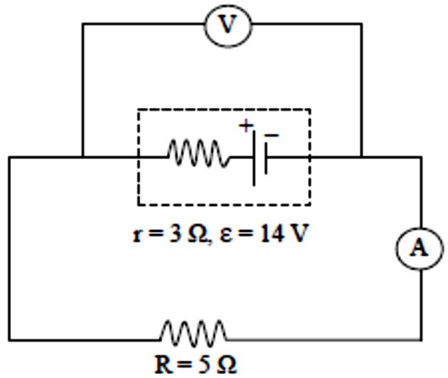


Diagram 2



(i) Determine the readings of the idealised voltmeter V and ammeter A in **Diagram 1**. **(1 mark)**
(ii) What are the readings of the voltmeter V and ammeter A in **Diagram 2**? **(2 marks)**

FSFE 2013

QUESTION 3

A metal wire of cross-sectional area 2.0 mm^2 carries a current of 3.0 A and has 5.0 x 10^{28} free electrons per cubic metre.

Calculate:

(i) its current density. **(1 mark)**

(ii) the drift velocity of the electrons. **(2 marks)**

QUESTION 5

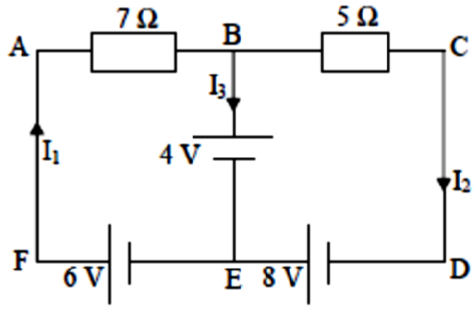
(a) An electricity supply cable consists of a steel core of cross-sectional area 50 mm^2 with six other aluminium conductors of the same cross-sectional area arranged around it.

[Resistivities: steel = 9.0 x 10^{-6} Ωm, aluminium = 2.5 x 10^{-8} Ωm]

(i) Find the resistance of a 120 m length steel cable. **(1 mark)**

(ii) Find the effective resistance of the supply cable 120 m long. **(2 marks)**

(c) A multi-loop circuit is given below.



(i) Using junction B, write the equation relating the currents I_1 , I_2 and I_3. **(1 mark)**

(ii) Using Kirchoff's Law, write the equation for loops:

I. ABEFA

II. ACDFA

(1 mark)

i)
ii) I. ABEFA II. ACDFE