Sangam S. K. M College - Nadi

Year 13

Mathematics

Worksheet 1: Solution

1. **Solve** $9x^2 + 25 = 0,\ x \in z.$

   $\boldsymbol{9x^2 + 25 = 0}$

   $\boldsymbol{x^2 = \frac{-25}{9}}$

   $\boldsymbol{x = \sqrt{\frac{-25}{9}} = \sqrt{\frac{25}{9}}i}$

   $\boldsymbol{x = \{-\frac{5}{3}i, \frac{5}{3}i\}}$

2. **Find the values of *x* and y** in the equation: $x + yi = \frac{1}{3 - 4i}$

   $\boldsymbol{= \frac{1}{3-4i} \cdot \frac{3+4i}{3+4i}}$

   $\boldsymbol{= \frac{3+4i}{9 - 16i^2} = \frac{3+4i}{25}}$

   $\boldsymbol{x = \frac{3}{25}}$ **and** $\boldsymbol{y = \frac{4}{25}}$

3. If v = 2 + 3i and w = 5 + 4i , **find**:

   a) v + w

   $= \boldsymbol{(2 + 3i) + (5 + 4i)}$

   $= \boldsymbol{7 + 7i}$

   b) w – v

   $= \boldsymbol{(5 + 4i) - (2 + 3i)}$

   $= \boldsymbol{3 + i}$

   c) $\bar{v}$

   $= \boldsymbol{2 - 3i}$

4. A complex number is given as w = $\sqrt{12} + 2i$

a) Find $|w|$

$|w|$ = 4

b) Find Arg (w)

Arg (w) = 30^0

c) Convert w into polar form.

w = 4 cis 30^0

d) Hence, evaluate w^3 using De Moivre's Theorem.

w^3 = $\mathbf{4^3}$ **cis** $(\mathbf{3})(\mathbf{30^0})$

$= \mathbf{64\ cis\ 90^0}$

5. **Solve** the equation z^2 = 64 (cos 90° + i sin 90°).

Express your answer in **rectangular form**.

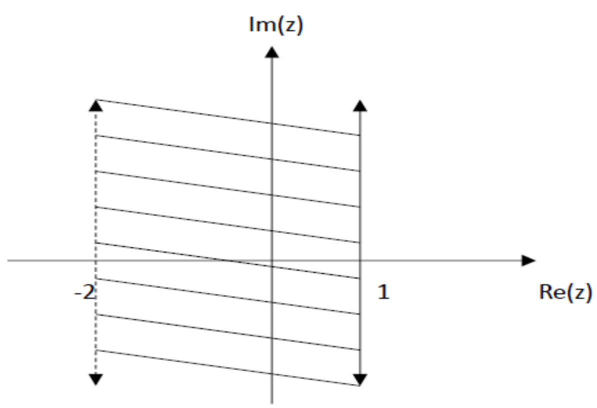
$\mathbf{64^{\frac{1}{2}} = 8}$ $\qquad \frac{90°}{\mathbf{2}} = \mathbf{45^0}$ $\qquad \frac{360°}{\mathbf{2}} = \mathbf{180^0}$

$\boldsymbol{w_0}$ **= 8 cis $\mathbf{45^0}$ = $\sqrt{\mathbf{32}}$ + $\sqrt{\mathbf{32}}\ \boldsymbol{i}$**

$\boldsymbol{w_1}$ **= 8 cis $\mathbf{225^0}$ = - $\sqrt{\mathbf{32}}$ - $\sqrt{\mathbf{32}}\ \boldsymbol{i}$**

$= \{\sqrt{\mathbf{32}} + \sqrt{\mathbf{32}}\ \boldsymbol{i}, -\sqrt{\mathbf{32}} - \sqrt{\mathbf{32}}\ \boldsymbol{i}\}$

6. In the complex plane, **shade** the region where -2 < Re (z) ≤ 1.



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Solution: Week 1

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Activity: Year 13 Mathematics textbook page 131: Exercise 6.1.1 - Questions 1, 3, 5

1. A sequence $<a_n>$ is defined by $a_n = \frac{7n+3}{n-9}$

   a) Find the first four terms of the sequence.

   $= \left\langle \frac{7(1)+3}{1-9}, \frac{7(2)+3}{2-9}, \frac{7(3)+3}{3-9}, \frac{7(4)+3}{4-9}, \ldots \right\rangle$

   $= \left\langle \frac{10}{-8}, \frac{17}{-7}, \frac{24}{-6}, \frac{31}{-5}, \ldots \right\rangle$

   $< a_n > = \left\langle -\frac{5}{4}, -\frac{17}{7}, -4, -\frac{31}{5}, \ldots \right\rangle$

   b) Find the first three terms of the sequence of partial sums.

   $\boldsymbol{S_1} = -\frac{5}{4}$

   $\boldsymbol{S_2} = -\frac{5}{4} + -\frac{17}{7} = -\frac{103}{28}$

   $\boldsymbol{S_3} = -\frac{103}{28} + -4 = -\frac{215}{28}$

   $< \boldsymbol{S_n} > = \left\langle -\frac{5}{4}, -\frac{103}{28}, -\frac{215}{28}, \ldots \right\rangle$

   c) Find $\lim_{n \to \infty} \frac{7n+3}{n-9}$

   $\lim_{n \to \infty} \frac{7n+3}{n-9}$ ***= 7 (using L'Hopital's Rule)***

   d) Explain why the sequence converges.

   sequence has a limit thus the sequence converges to 7.

3. A sequence <a_n> is defined by $a_n = \frac{n+2}{n^2}$

a) Find the first two terms of the sequence of partial sums.

$\mathbf{T_1} = \frac{\mathbf{1+2}}{\mathbf{1^2}} = \boldsymbol{3}$

$\boldsymbol{T_2} = \frac{\mathbf{2+2}}{\mathbf{2^2}} = \boldsymbol{1}$

$\boldsymbol{S_1} = \boldsymbol{3}$

$\boldsymbol{S_2} = \boldsymbol{3} + \mathbf{1} = \boldsymbol{4}$

$< \boldsymbol{S_n} > \; = \; \langle \mathbf{3}, \;\; \mathbf{1}, \ldots \rangle$

b) Determine whether a sequence converges or diverges, and if it converges, give the value to which it converges to.

$= \lim_{n \to \infty} \frac{n+2}{n^2}$

$= 0$

thus the sequence converges to 0

5. A sequence <a_n> is defined by $a_n =$ 3n + 4
Does it converge or diverge. Explain

$= \lim_{n \to \infty} 3n + 4$

$= 3(\infty) + 4$

$= \infty$

The limit is infinity thus the sequence diverges